SECOND FUNDAMENTAL THEOREM OF CALCULUS

Consider the function $f(x) = \sin(x^2)$. We would like to find an antiderivative, F, of the function f. However, there is no way to describe F in terms of known elementary functions. However, from the graph of f, we can certainly obtain a graph of F. The fundamental Theorem of Calculus gives a way to express antiderivatives of given functions.

For example: $F(x) = \displaystyle\int_0^x \sin(t^2)\,dt$ is a *specific* antiderivative of a continuous function f. This particular F satisfies $F(0) = 0$. Note that the independent variable of the function F is x which appears as the upper limit of integration. One can use any variable inside the integral sign.

More generally, the function F described by $F(x) = \displaystyle\int_a^x f(t)\,dt$ is **the** antiderivative of f which satisfies $F(a) = 0$.

So what do we do if we want to construct the antiderivative of $f(x) = e^{-x^2}$ passing through the point $(3, 105)$? This would be accomplished by the writing

$$F(x) = \int_3^x e^{-t^2}\,dt + 105$$

Note that $F'(x) = e^{-x^2}$ and $F(3) = \int_3^3 e^{-t^2}\,dt + 105 = 105$ as required.

Of course, we can now combine F with other known functions to construct more complex functions. For example if $F(x) = \displaystyle\int_a^x f(t)\,dt$, and h is another function, then we can construct the composite function $G(x) = F(h(x)) = \displaystyle\int_a^{h(x)} f(t)\,dt$. Of course, if we want to differentiate G, we must use the chain rule.