

MA 1122/1422

Polytechnic University
SAMPLE EXAM 1

SPRING 2005

EXAM 1: _____

EXAM 2: _____

GRADE: _____

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last page contains formulas that you might find useful. You may tear that page out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

YOUR SIGNATURE:

- (1) (Page 359, Problem 28) The circumference of a tree at different heights above the ground is given in the table below. Assume that all horizontal cross-sections of the tree are circles.

Height (inches)	0	20	40	60	80	100	120
Circumference (inches)	31	28	21	17	12	8	2

Use the Simpson's rule to estimate the volume of the tree. Clearly indicate the number of subdivisions and show all your work.

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- (2) Determine whether each of the following improper integrals converges or diverges. For each convergent integral, find its *exact value*. Show all your work.

(a) (Pages 332, Example 7) $\int_0^7 \frac{1}{(x-4)^{2/3}} dx$

(b) (Page 333, Problem 30) $\int_7^{\infty} \frac{1}{\sqrt{y-4}} dy$

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- (3) (Page 340, Problems 127–140) Determine whether the following integral converges. If it converges, find an upper bound for the value of the integral.

$$\int_0^{\pi/4} \frac{\sin(2x)dx}{(\cos(2x) + \cos^5(2x))^{1/3}}.$$

Show all your work.

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(4) (Concepts 7.7–7.8) Decide whether each of the following statements is TRUE or FALSE. (You do not have to explain.)

(a) If $f(x)$ is continuous for all x and $\int_0^\infty f(x) dx$ converges, then so does $\int_a^\infty f(x) dx$ for all positive a .

(b) If $f(x)$ is continuous and positive for $x > 0$ and if $\lim_{x \rightarrow \infty} f(x) = 0$, then $\int_1^\infty f(x) dx$ converges.

(c) If p is a positive integer such that $\int_0^{10} \frac{1}{x^p} dx$ converges then $\int_0^{10} \frac{1}{\sqrt{x^p}} dx$ also converges.

(d) $\int_0^{10} \frac{1}{\sqrt{x^5 + x}} dx$ converges.

(e) If $\int_0^\infty f(x) dx$ and $\int_0^\infty g(x) dx$ both diverge, then $\int_0^\infty (f(x) + g(x)) dx$ diverges.

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- (5) (Page 350, Problems 1–8) Let \mathcal{R} be the region bounded by the two curves $y^2 = x$ and $2y = x - 3$. Which of the following expressions gives the area of \mathcal{R} ? Circle **all** correct choices. You do not have to explain.

(a) $\int_0^9 \sqrt{x} - \frac{x-3}{2} dx$

(b) $\int_{-1}^3 2y + 3 - y^2 dy$

(c) $\int_0^1 2\sqrt{x} dx + \int_1^9 \sqrt{x} - \frac{x-3}{2} dx$

(d) $\int_0^9 \left| \sqrt{x} - \frac{x-3}{2} \right| dx$

(e) $\int_0^9 2y + 3 - y^2 dy$

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- (6) (Page 358, Problem 21) Let \mathcal{R} be the region bounded by the curve $y = e^x$, the x -axis, and the lines $x = 0$, $x = 1$. **Set up**, but do not evaluate, the definite integral that gives the volume of the solid obtained by rotating \mathcal{R} about the horizontal line $y = -3$.

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- (7) (Page 358, Problem 18) Let \mathcal{R} be the region bounded by the curve $y = 2^x$, the line $y + 2x = 4$, and the y -axis. **Set up**, but do not evaluate, the definite integral which gives the volume of the solid obtained by rotating \mathcal{R} about the y -axis. Show all your work.

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- (8) (Page 359, Problem 33) Let $f(x) = (e^{2x} + e^{-2x})/4$. Find the arc length of this function between $x = -3$ and $x = 3$. Show all your work.

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(9) (Page 368, Problem 20) A rod of length 1 meter has the density $\delta(x) = 1 + kx^2$ grams/meter, where k is a positive constant. The rod is lying on the positive x -axis with one end at the origin.

(a) Find the center of the mass as a function of k . Show all your work.

(b) Show that the center of mass of the rod is between $x = 0.5$ to $x = 0.75$ on the x -axis. Show all your work.

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- (10) (Page 367, Problem 9) Find the total mass of the triangular region with its vertices at $(-1, 0)$, $(1, 0)$, and $(0, 2)$, (x and y in cm) and with density $\delta(x) = 1 + x$ grams/cm². Show all your work.

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Useful formulas

- *Physics formulas:*

The *acceleration* due to gravity, g : $g = 9.8\text{m/sec}^2$, or $g = 32\text{ft/sec}^2$.

Mass density of water = 1000 kg/m^3 , Weight density of water = 62.4 lbs/ft^3 .

Force = mass \times acceleration Work = Force \times distance

The center of mass, \bar{x} , of an object lying on the x -axis between $x = a$ and $x = b$,

with mass density $\delta(x)$ is given by $\bar{x} = \frac{\int_a^b x\delta(x) dx}{\text{total mass}}$

The center of mass, \bar{x} , of n **discrete** masses m_i lying along the x -axis, each located

at x_i is given by $\bar{x} = \frac{\sum_{i=1}^n x_i m_i}{\sum_{i=1}^n m_i}$

Arc length of a curve $y = f(x)$ from $x = a$ to $x = b$: $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$

- *Integration by Parts:*

$$\int u dv = uv - \int v du \quad \text{or} \quad \int uv' dx = uv - \int vu' dx$$

- *Numerical Approximations:*

$$\text{TRAP}(n) = \frac{\text{LEFT}(n) + \text{RIGHT}(n)}{2}; \quad \text{SIMP}(n) = \frac{2\text{MID}(n) + \text{TRAP}(n)}{3}$$

- *Useful Integrals for Comparison:*

$\int_1^\infty \frac{1}{x^p} dx$ converges for $p > 1$ and diverges for $p \leq 1$.

$\int_0^1 \frac{1}{x^p} dx$ converges for $p < 1$ and diverges for $p \geq 1$.

$\int_0^\infty e^{-ax} dx$ converges for $a > 0$.

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

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Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

$$\begin{array}{l} 1. \int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad (n \neq -1) \\ 2. \int \frac{1}{x} dx = \ln|x| + C \\ 3. \int a^x dx = \frac{1}{\ln a}a^x + C \\ 4. \int \ln x dx = x \ln x - x + C \end{array} \quad \left\| \begin{array}{l} 5. \int \sin ax dx = -\frac{1}{a} \cos ax + C \\ 6. \int \cos ax dx = \frac{1}{a} \sin ax + C \\ 7. \int \tan ax dx = -\frac{1}{a} \ln|\cos ax| + C \end{array} \right.$$

II. Products of e^x , $\cos x$, and $\sin x$

$$\begin{array}{l} 8. \int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C \\ 9. \int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C \\ 10. \int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 11. \int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 12. \int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b \end{array}$$

III. Product of Polynomial $p(x)$ with $\ln x, e^x$, $\cos x$, and $\sin x$

$$\begin{array}{l} 13. \int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1, x > 0 \\ 14. \int p(x) e^{ax} dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \dots + C \\ \quad (+ - + - + - + \dots) \text{ (signs alternate)} \\ 15. \int p(x) \sin ax dx = -\frac{1}{a} p(x) \cos(ax) + \frac{1}{a^2} p'(x) \sin(ax) + \frac{1}{a^3} p''(x) \cos(ax) - \dots + C \\ \quad (- + + - - + + - \dots) \text{ (signs alternate in pairs)} \\ 16. \int p(x) \cos ax dx = \frac{1}{a} p(x) \sin(ax) + \frac{1}{a^2} p'(x) \cos(ax) - \frac{1}{a^3} p''(x) \sin(ax) - \dots + C \\ \quad (+ + - - + + - \dots) \text{ (signs alternate in pairs)} \end{array}$$

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IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \quad n \text{ positive}$
18. $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx, \quad n \text{ positive}$
19. $\int \frac{1}{\sin^m x} \, dx = -\frac{1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$
20. $\int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$
21. $\int \frac{1}{\cos^m x} \, dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$
22. $\int \frac{1}{\cos x} \, dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$
23. $\int \sin^m x \cos^n x \, dx :$

If n is odd, let $w = \sin x$.

If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18.

If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.

The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C, \quad a \neq 0$
25. $\int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + \frac{c}{a} \arctan \left(\frac{x}{a} \right) + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} (\ln |x-a| - \ln |x-b|) + C, \quad a \neq b$
27. $\int \frac{cx + d}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} [(ac + d) \ln |x-a| - (bc + d) \ln |x-b|] + C, \quad a \neq b$

VI. Integrands involving $\sqrt{a^2 + x^2}, \sqrt{a^2 - x^2}, \sqrt{x^2 - a^2}, a > 0$

28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \left(\frac{x}{a} \right) + C$
29. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C$
30. $\int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left(x\sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C$
31. $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left(x\sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C$