

MA 1122/1422

Polytechnic University
EXAM 1

APRIL 18, 2005

EXAM 1: _____

EXAM 2: _____

GRADE: _____

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last few pages contain formulas that you might find useful. You may tear those pages out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	14	
2	14	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
Total	100	

YOUR SIGNATURE:

(1) (Page 333, Problems 5–32) In each part, circle the correct choice. You need not show any work.

(a) $\int_0^{\infty} te^{-t^2} dt =$

- (i) $1/2$.
- (ii) 1 .
- (iii) $-1/2$.
- (iv) -1 .
- (v) The integral diverges.

(b) $\int_{-1}^8 \frac{2}{\sqrt[3]{x^4}} dx =$

- (i) -9 .
- (ii) 3 .
- (iii) -18 .
- (iv) -6 .
- (v) The integral diverges.

YOUR SIGNATURE:

(2) (Page 333, Problems 5–32) In each part, circle the correct choice. You need not show any work.

(a) $\int_0^{\infty} \frac{dz}{z^2 + 16} =$

- (i) $\pi/2$.
- (ii) $\pi/8$.
- (iii) $\pi/32$.
- (iv) $1/4$.
- (v) The integral diverges.

(b) $\int_{2e}^{\infty} \frac{1}{x(\ln x)^2} dx =$

- (i) $\frac{1}{2}$.
- (ii) $\frac{-1}{2e}$.
- (iii) $\frac{1}{2e}$.
- (iv) $\frac{1}{1 + \ln 2}$.
- (v) The integral diverges.

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- (3) (Page 337, Problem 24) Determine whether the following integral converges. If it converges, find an upper bound for the value of the integral.

$$\int_1^5 \frac{5 - 2 \sin(2x)}{\sqrt{x - 1}} dx.$$

Show all your work.

YOUR SIGNATURE:

- (4) (Sample Exam) Let \mathcal{R} be the region enclosed by $y = |x - 2|$ and $y = 4 - x^2$. Find the volume of the solid obtained by rotating \mathcal{R} about the line $y = -1$. Show your work.

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- (5) (Page 358, Problem 11) Find the exact length of the curve

$$y = \frac{4\sqrt{2}}{3}x^{3/2} - 1 \quad \text{for} \quad 0 \leq x \leq 1.$$

Show all your work.

YOUR SIGNATURE:

- (6) (Sample Exam) An electrician suspects that a meter showing the total consumption Q in kilowatt hours (kWh) of electricity is not functioning properly. To check the accuracy, the electrician measures the consumption rate R directly every 10 minutes obtaining the following table.

t (min)	0	10	20	30	40	50	60
R (kWh/min)	1.31	1.43	1.45	1.39	1.36	1.47	1.29

- (a) Write a definite integral which gives the total consumption during the one-hour period.
- (b) Give the best estimate of the total consumption during this one-hour period using Simpson's rule. State clearly the number of subdivisions. Show all your work.

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(7) (Concepts 7.7–7.8) Decide whether each of the following statements is TRUE or FALSE. You do not have to explain.

(a) If $f(x)$ is a continuous function for all x and $\int_0^\infty f(x) dx$ converges, then so does $\int_0^\infty f(x - a) dx$ for all positive constant a .

(b) If $f(x)$ is a continuous function for all x and $\int_0^\infty f(x) dx$ converges, then so does $\int_0^\infty f(x + a) dx$ for all positive constant a .

(c) If $f(x)$ is a continuous function for all x and $\int_0^\infty f(x) dx$ converges, then so does $\int_0^\infty (a + f(x)) dx$ for all positive constant a .

(d) If $f(x)$ is a continuous function for all x and $\int_0^\infty f(x) dx$ converges, then so does $\int_0^\infty f(ax) dx$ for all positive constant a .

(e) If $\int_0^\infty f(x) dx$ diverges and $f(x) \geq g(x)$ for all x , then $\int_0^\infty g(x) dx$ also diverges.

YOUR SIGNATURE:

(8) (Page 368, Problem 22) Suppose that a plate of uniform density 3 gm/cm^2 is placed in the xy -plane such that its shape is the region bounded by the parabola $x = 4 - y^2$ and the y -axis, with x and y measured in cm.

(a) Find the total mass of the plate. Show all your work.

(b) Set up, but do not evaluate, the integral, which gives x -coordinate of its center of mass. Show all your work.

YOUR SIGNATURE: _____

Useful formulas

- *Physics formulas:*

The *acceleration* due to gravity, g : $g = 9.8\text{m/sec}^2$, or $g = 32\text{ft/sec}^2$.

Mass density of water = 1000 kg/m^3 , Weight density of water = 62.4 lbs/ft^3 .

Force = mass \times acceleration Work = Force \times distance

The center of mass, \bar{x} , of an object lying on the x -axis between $x = a$ and $x = b$,

with mass density $\delta(x)$ is given by $\bar{x} = \frac{\int_a^b x\delta(x) dx}{\text{total mass}}$

The center of mass, \bar{x} , of n **discrete** masses m_i lying along the x -axis, each located

at x_i is given by $\bar{x} = \frac{\sum_{i=1}^n x_i m_i}{\sum_{i=1}^n m_i}$

Arc length of a curve $y = f(x)$ from $x = a$ to $x = b$: $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$

- *Integration by Parts:*

$$\int u dv = uv - \int v du \quad \text{or} \quad \int uv' dx = uv - \int vu' dx$$

- *Numerical Approximations:*

$$\text{TRAP}(n) = \frac{\text{LEFT}(n) + \text{RIGHT}(n)}{2}; \quad \text{SIMP}(n) = \frac{2\text{MID}(n) + \text{TRAP}(n)}{3}$$

- *Useful Integrals for Comparison:*

$\int_1^\infty \frac{1}{x^p} dx$ converges for $p > 1$ and diverges for $p \leq 1$.

$\int_0^1 \frac{1}{x^p} dx$ converges for $p < 1$ and diverges for $p \geq 1$.

$\int_0^\infty e^{-ax} dx$ converges for $a > 0$.

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

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Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

$$\begin{array}{l} 1. \int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad (n \neq -1) \\ 2. \int \frac{1}{x} dx = \ln|x| + C \\ 3. \int a^x dx = \frac{1}{\ln a}a^x + C \\ 4. \int \ln x dx = x \ln x - x + C \end{array} \quad \left\| \begin{array}{l} 5. \int \sin ax dx = -\frac{1}{a} \cos ax + C \\ 6. \int \cos ax dx = \frac{1}{a} \sin ax + C \\ 7. \int \tan ax dx = -\frac{1}{a} \ln|\cos ax| + C \end{array} \right.$$

II. Products of e^x , $\cos x$, and $\sin x$

$$\begin{array}{l} 8. \int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C \\ 9. \int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C \\ 10. \int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 11. \int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 12. \int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b \end{array}$$

III. Product of Polynomial $p(x)$ with $\ln x, e^x$, $\cos x$, and $\sin x$

$$\begin{array}{l} 13. \int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1, x > 0 \\ 14. \int p(x) e^{ax} dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \dots + C \\ \quad (+ - + - + - + \dots) \text{ (signs alternate)} \\ 15. \int p(x) \sin ax dx = -\frac{1}{a} p(x) \cos(ax) + \frac{1}{a^2} p'(x) \sin(ax) + \frac{1}{a^3} p''(x) \cos(ax) - \dots + C \\ \quad (- + + - - + + - \dots) \text{ (signs alternate in pairs)} \\ 16. \int p(x) \cos ax dx = \frac{1}{a} p(x) \sin(ax) + \frac{1}{a^2} p'(x) \cos(ax) - \frac{1}{a^3} p''(x) \sin(ax) - \dots + C \\ \quad (+ + - - + + - \dots) \text{ (signs alternate in pairs)} \end{array}$$

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IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \quad n \text{ positive}$
18. $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx, \quad n \text{ positive}$
19. $\int \frac{1}{\sin^m x} \, dx = -\frac{1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$
20. $\int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$
21. $\int \frac{1}{\cos^m x} \, dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$
22. $\int \frac{1}{\cos x} \, dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$
23. $\int \sin^m x \cos^n x \, dx :$

If n is odd, let $w = \sin x$.

If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18.

If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.

The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C, \quad a \neq 0$
25. $\int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + \frac{c}{a} \arctan \left(\frac{x}{a} \right) + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} (\ln |x-a| - \ln |x-b|) + C, \quad a \neq b$
27. $\int \frac{cx + d}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} [(ac + d) \ln |x-a| - (bc + d) \ln |x-b|] + C, \quad a \neq b$

VI. Integrands involving $\sqrt{a^2 + x^2}, \sqrt{a^2 - x^2}, \sqrt{x^2 - a^2}, a > 0$

28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \left(\frac{x}{a} \right) + C$
29. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C$
30. $\int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left(x\sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C$
31. $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left(x\sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C$