

MA 1122/1422

Polytechnic University
EXAM 2

MAY 6, 2005

EXAM 1: _____

EXAM 2: _____

GRADE: _____

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last few pages contain formulas that you might find useful. You may tear those pages out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	12	
2	12	
3	10	
4	10	
5	10	
6	12	
7	12	
8	10	
9	12	
Total	100	

YOUR SIGNATURE:

(1) (Page 410, Problems 1–18) In each part, circle the correct choice. You need not show any work.

(a) $\sum_{n=1}^{\infty} \frac{1}{3^n} =$

- (i) 0.
- (ii) $1/2$.
- (iii) $2/3$.
- (iv) $3/2$.
- (v) The series diverges.

(b) $\sum_{n=0}^{\infty} \frac{2^{2n}}{3^n} =$

- (i) -3 .
- (ii) 3.
- (iii) $2/3$.
- (iv) 6.
- (v) The series diverges.

(c) $\sum_{n=1}^{10} \frac{3^{n+1}}{5^n} =$

- (i) $9/2$.
- (ii) $\frac{9\left(1 - \left(\frac{3}{5}\right)^{10}\right)}{2}$.
- (iii) $\frac{9\left(1 - \left(\frac{3}{5}\right)^{11}\right)}{2}$.

- (iv) 6.
- (v) The series diverges.
- (vi) None of the above.

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- (2) (Page 422, Problems 1–28) Determine whether each of the following series converges. Circle all convergent series. You do not have to explain.

(a)
$$\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!}$$

(b)
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

(c)
$$\sum_{n=0}^{\infty} \frac{n^2 - 3}{n^2 + 1}$$

(d)
$$\sum_{n=0}^{\infty} \frac{1}{e^{-n}}$$

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(3) (Page 428, Problem 21) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(2x - 1)^{2n}}{3^n}.$$

Show all your work.

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(4) (Page 445, Problem 21) Find the second-degree Taylor polynomial for

$$f(x) = \ln(1 + 2x)$$

for x near 1. Show all your work.

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(5) (Page 448, Problem 37) Solve exactly for the variable x . Show all your work.

$$x + 2x^2 + \frac{4x^3}{2!} + \frac{8x^4}{3!} + \cdots + \frac{2^n x^{n+1}}{n!} + \cdots = \frac{x}{4}$$

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(6) (Page 446, Problem 40) The Taylor series of a certain function $h(x)$ near $x = 0$ is

$$2 - \frac{2x^2}{3!} + \frac{2x^4}{5!} - \frac{2x^6}{7!} + \frac{2x^8}{9!} - \dots .$$

Assuming that the pattern continues in the series, fill in the blanks. You do not need to explain.

(a) The exact value of $h^{(100)}(0)$, the 100th derivative of $h(x)$ at $x = 0$, is _____.

(b) The exact value of $h^{(101)}(0)$, the 101st derivative of $h(x)$ at $x = 0$, is _____.

(c) $\lim_{x \rightarrow 0} \frac{h(x) - 2}{\cos(x) - 1} =$ _____.

(d) The function $h(x)$ has a _____(local minimum/maximum) at $x = 0$.

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(7) (Page 473, Problems 1–24; Page 945, Problems 37–42) Determine whether each of the statements below is TRUE or FALSE. No explanation required.

(a) If the third degree Taylor polynomial of a function $f(x)$ near $x = p$ is

$$5 + 2(x - p) - 7(x - p)^2 + (x - p)^3,$$

then $f(x)$ has a local maximum at $x = p$.

(b) The Taylor series of $h(x) = f(x)g(x)$ near $x = 0$ is

$$f(0)g(0) + f'(0)g'(0)x + \frac{f''(0)g''(0)}{2!}x^2 + \frac{f'''(0)g'''(0)}{2!}x^2 + \dots$$

(c) If $g^{(n)}(0) \geq n!$ for all n , then the Taylor series for g near $x = 0$ diverges at $x = 1$.

(d) The Taylor series for $f(x) = x \cos(x)$ about $x = 0$ has only odd powers.

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- (8) (Page 452, Problem 27) Assume R is a positive constant. Suppose V is given by the expression

$$V = \sqrt{R^2 + a^2} - R.$$

Expand V as a series in a as far as the third nonzero term. Show all your work.

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(9) (Page 411, Problems 20–21) The quantity of the drug ampicillin in the blood decays exponentially as time goes by. Ampicillin is taken in 250 mg doses once every 12 hours to treat bacterial infections.

(a) If the half-life of ampicillin is 3 hours, what percentage of the ampicillin present at the start of a 12-hour period is still there at the end? Show all your work.

(b) If P_n denotes the quantity of ampicillin (in mg) in the body right **before** the n th tablet is taken, then what is $\lim_{n \rightarrow \infty} P_n$? Show all your work.

Useful formulas

- *Geometry Formulas*

Here V is the volume, S is the surface area, h is the height and r is the radius.

Cylinder with top and bottom: $V = \pi r^2 h$, $S = 2\pi r h + 2\pi r^2$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$

- *Physics formulas:*

The *acceleration* due to gravity, g : $g = 9.8\text{m/sec}^2$, or $g = 32\text{ft/sec}^2$.

Mass density of water = 1000 kg/m^3 , Weight density of water = 62.4 lbs/ft^3 .

Force = mass \times acceleration Work = Force \times distance

The center of mass, \bar{x} , of an object lying on the x -axis between $x = a$ and $x = b$,

with mass density $\delta(x)$ is given by $\bar{x} = \frac{\int_a^b x\delta(x) dx}{\text{total mass}}$

Arc length of a curve $y = f(x)$ from $x = a$ to $x = b$: $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$

- *Integration by Parts:*

$$\int u dv = uv - \int v du \quad \text{or} \quad \int uv' dx = uv - \int vu' dx$$

- *Numerical Approximations:*

$$\text{TRAP}(n) = \frac{\text{LEFT}(n) + \text{RIGHT}(n)}{2}; \quad \text{SIMP}(n) = \frac{2\text{MID}(n) + \text{TRAP}(n)}{3}$$

- *Finite Geometric Sum:*

$$a + ax + ax^2 + \cdots + ax^{n-1} = \frac{a(1 - x^n)}{1 - x}$$

- *Infinite Geometric Series:*

$$a + ax + ax^2 + \cdots = \frac{a}{1 - x} \quad \text{for } |x| < 1$$

- *Ratio Test:*

For the series $\sum a_n$, suppose,

$$\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = L.$$

- If $L < 1$, then the series converges.
- If $L > 1$, then the series diverges.
- If $L = 1$, then the test fails.

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- n th degree Taylor Polynomial of $f(x)$ centered at $x = a$:

$$f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x - a)^n$$

- Taylor series of $f(x)$ centered at $x = a$:

$$f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \dots$$

- Taylor Series of important functions:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \dots \quad \text{for } -1 < x < 1$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad \text{for } -1 < x \leq 1$$

$$(1 + x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots \quad \text{for } -1 < x < 1$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

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Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

$$\begin{array}{l} 1. \int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad (n \neq -1) \\ 2. \int \frac{1}{x} dx = \ln|x| + C \\ 3. \int a^x dx = \frac{1}{\ln a}a^x + C \\ 4. \int \ln x dx = x \ln x - x + C \end{array} \quad \left\| \begin{array}{l} 5. \int \sin ax dx = -\frac{1}{a} \cos ax + C \\ 6. \int \cos ax dx = \frac{1}{a} \sin ax + C \\ 7. \int \tan ax dx = -\frac{1}{a} \ln|\cos ax| + C \end{array} \right.$$

II. Products of e^x , $\cos x$, and $\sin x$

$$\begin{array}{l} 8. \int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C \\ 9. \int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C \\ 10. \int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 11. \int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 12. \int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b \end{array}$$

III. Product of Polynomial $p(x)$ with $\ln x, e^x$, $\cos x$, and $\sin x$

$$\begin{array}{l} 13. \int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1, x > 0 \\ 14. \int p(x) e^{ax} dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \dots + C \\ \quad (+ - + - + - + \dots) \text{ (signs alternate)} \\ 15. \int p(x) \sin ax dx = -\frac{1}{a} p(x) \cos(ax) + \frac{1}{a^2} p'(x) \sin(ax) + \frac{1}{a^3} p''(x) \cos(ax) - \dots + C \\ \quad (- + + - - + + - - \dots) \text{ (signs alternate in pairs)} \\ 16. \int p(x) \cos ax dx = \frac{1}{a} p(x) \sin(ax) + \frac{1}{a^2} p'(x) \cos(ax) - \frac{1}{a^3} p''(x) \sin(ax) - \dots + C \\ \quad (+ + - - + + - - \dots) \text{ (signs alternate in pairs)} \end{array}$$

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IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \quad n \text{ positive}$
18. $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx, \quad n \text{ positive}$
19. $\int \frac{1}{\sin^m x} \, dx = -\frac{1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$
20. $\int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$
21. $\int \frac{1}{\cos^m x} \, dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$
22. $\int \frac{1}{\cos x} \, dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$
23. $\int \sin^m x \cos^n x \, dx :$

If n is odd, let $w = \sin x$.

If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18.

If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.

The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C, \quad a \neq 0$
25. $\int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + \frac{c}{a} \arctan \left(\frac{x}{a} \right) + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} (\ln |x-a| - \ln |x-b|) + C, \quad a \neq b$
27. $\int \frac{cx + d}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} [(ac + d) \ln |x-a| - (bc + d) \ln |x-b|] + C, \quad a \neq b$

VI. Integrands involving $\sqrt{a^2 + x^2}, \sqrt{a^2 - x^2}, \sqrt{x^2 - a^2}, a > 0$

28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \left(\frac{x}{a} \right) + C$
29. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C$
30. $\int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left(x\sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C$
31. $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left(x\sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C$