Polytechnic University

MA 1122/1132 EXAM 1 FEBRUARY 10, 2006

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last few pages contain formulas that you might find useful. You may tear those pages out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	15	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	15	
Total	100	

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(1) (Page 333, Problems 5–32) In each part, circle the correct choice. You need not show any work.

(a) $\displaystyle\int_0^\pi \frac{1}{\sqrt{x}}\, e^{-\sqrt{x}}\, dx =$

(i) $\frac{1}{2} - \frac{1}{2}e^{-\sqrt{\pi}}$.

(ii) $2(1 - e^{-\sqrt{\pi}})$.

(iii) $2e^{-\sqrt{\pi}} - 2$.

(iv) $\dfrac{1}{\sqrt{\pi}}$.

(v) The integral diverges.

(vi) None of the above.

(b) $\displaystyle\int_1^2 \frac{dx}{x \ln x} =$

(i) $\ln(2)$.

(ii) $(\ln(2))^2 - 1$.

(iii) $\ln(\ln(2))$.

(iv) $\ln(\ln(2)) - 1$.

(v) The integral diverges.

(vi) None of the above.

(c) $\displaystyle\int_2^\infty \frac{t^2}{\sqrt{t^3 - 4}}\, dt =$

(i) $\dfrac{4}{3}$.

(ii) $-\dfrac{1}{3}$.

(iii) -1.

(iv) 2.

(v) The integral diverges.

(vi) None of the above.

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(2) (Page 326, Problem 5) Estimate the value of the integral $\int_0^2 \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx$ using Simpson's rule with 4 subintervals. Show all your work.

(3) (Page 337, Problem 25) Determine whether the following integral converges. If it converges, find an upper bound for the value of the integral.

$$\int_0^{\infty} \frac{3\sin(2x) - 6}{4x^2 + 1}\, dx$$

Show all your work.

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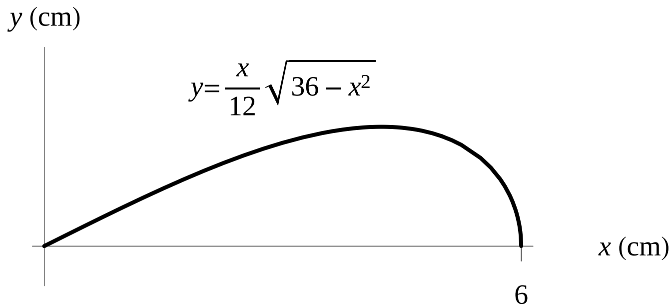
(4) (Page 358, Problem 22; Sample Exam) A plate with uniform density 3 gm/cm^2 is bounded by the curve $y = x^2 + 2x + 1$ and the line $y - x = 3$.

(a) Find the total mass of the plate. Show all your work.

(b) Find the x-coordinate of its center of mass, $\bar{x}$. Show all your work.

(5) (Page 368, Problem 22) Let $\mathcal{R}$ be the region bounded by $y = \dfrac{x}{12}\sqrt{36 - x^2}$ and $y = 0$, as depicted in the figure below. **Set up**, but do not evaluate, the definite integral which gives the volume of the solid obtained by rotating $\mathcal{R}$ about the x-axis. Show all your work.



(6) (Page 358, Problem 12) Find the exact length of the arc described by $y = 1 - x^{3/2}$ from $x = 1$ to $x = 2$. Show all your work.

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(7) (Page 337, Problems 1–25) Determine which of the following integrals converges. Circle all convergent integrals. You need not show any work.

(a) $\displaystyle\int_1^\infty \frac{x^2}{e^{-x}+x}\,dx$

(b) $\displaystyle\int_{-1}^0 \frac{dx}{\sqrt{1+x}}$

(c) $\displaystyle\int_0^\infty \frac{e^{-z}}{1+z}\,dz$

(d) $\displaystyle\int_3^\infty \frac{t^3}{\sqrt{t^7-2}}\,dt$

(e) $\displaystyle\int_{-4}^\infty \frac{t^2-1}{t^3+2t+1}\,dt$

(8) (Page 429, Problem 16)

(a) Write the repeating decimal $0.1414141414\ldots$ as a geometric series.

(b) Use your answer in part (a) to write $0.1414141414\ldots$ as a fraction with an integer for the numerator and an integer for the denominator. Show your work.

(9) (Page 416, Problems 13–19) Fill in the blanks.

(a) The series $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$ is ______________________(convergent/divergent).

If it is convergent, then the sum is ______________________.

(b) The series $\displaystyle\sum_{n=0}^{\infty} \frac{(-4)^n}{3^{2n}}$ is ______________________(convergent/divergent).

If it is convergent, then the sum is ______________________.

(c) The series $\displaystyle\sum_{n=0}^{\infty} \frac{10^n + 4^n}{3^{2n}}$ is ______________________(convergent/divergent).

If it is convergent, then the sum is ______________________.

Useful formulas

- *Physics formulas:*
The *acceleration* due to gravity, g: $g = 9.8\text{m/sec}^2$, or $g = 32\text{ft/sec}^2$.
Mass density of water $= 1000\ \text{kg/m}^3$, Weight density of water $= 62.4\ \text{lbs/ft}^3$.
The center of mass, $\bar{x}$, of an object lying on the x-axis between $x = a$ and $x = b$, with mass density $\delta(x)$ is given by $\bar{x} = \dfrac{\int_a^b x\delta(x)\,dx}{\text{total mass}}$
The center of mass, $\bar{x}$, of n **discrete** masses m_i lying along the x-axis, each located at x_i is given by $\bar{x} = \dfrac{\sum_{i=1}^n x_i m_i}{\sum_{i=1}^n m_i}$
Arc length of a curve $y = f(x)$ from $x = a$ to $x = b$: $L = \displaystyle\int_a^b \sqrt{1 + (f'(x))^2}\,dx$
- *Integration by Parts*:

$$\int u dv = uv - \int v du \qquad \text{or} \qquad \int uv' dx = uv - \int vu' dx$$

- *Numerical Approximations:*

$$\text{TRAP}(n) = \frac{\text{LEFT}(n)+\text{RIGHT}(n)}{2}; \qquad \text{SIMP}(n) = \frac{2\text{MID}(n)+\text{TRAP}(n)}{3}$$

- Finite Geometric Sum:

$$a + ax + ax^2 + \cdots + ax^{n-1} = \frac{a(1-x^n)}{1-x}$$

- Infinite Geometric Series:

$$a + ax + ax^2 + \cdots = \frac{a}{1-x} \qquad \text{for } |x| < 1$$

- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln\|x\|) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

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Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C,\ (n \neq -1)$
2. $\int \frac{1}{x}\ dx = \ln|x| + C$
3. $\int a^x\ dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\ dx = x\ln x - x + C$
5. $\int \sin ax\ dx = -\frac{1}{a}\cos ax + C$
6. $\int \cos ax\ dx = \frac{1}{a}\sin ax + C$
7. $\int \tan ax\ dx = -\frac{1}{a}\ln|\cos ax| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax}\sin(bx)\ dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\int e^{ax}\cos(bx)\ dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\int \sin(ax)\sin(bx)\ dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax)\cos(bx)\ dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax)\cos(bx)\ dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, and $\sin x$

13. $\int x^n \ln x\ dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, x > 0$
14. $\int p(x)e^{ax}\ dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots + C$

    $(+-+-+-+\ldots)$ (signs alternate)
15. $\int p(x)\sin ax\ dx = -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a^2}p'(x)\sin(ax) + \frac{1}{a^3}p''(x)\cos(ax) - \cdots + C$

    $(-++--++--\ldots)$ (signs alternate in pairs)
16. $\int p(x)\cos ax\ dx = \frac{1}{a}p(x)\sin(ax) + \frac{1}{a^2}p'(x)\cos(ax) - \frac{1}{a^3}p''(x)\sin(ax) - \cdots + C$

    $(++--++--\ldots)$ (signs alternate in pairs)

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IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx, \quad n \text{ positive}$

18. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx, \quad n \text{ positive}$

19. $\int \frac{1}{\sin^m x}\,dx = -\frac{1}{m-1}\frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2} x}\,dx, \quad m \neq 1, m \text{ positive}$

20. $\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C$

21. $\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2} x}\,dx, \quad m \neq 1, m \text{ positive}$

22. $\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{\sin x + 1}{\sin x - 1}\right| + C$

23. $\int \sin^m x\cos^n x\,dx:$
If n is odd, let $w = \sin x$.
If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18.
If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.
The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C,\ a \neq 0$

25. $\int \frac{bx + c}{x^2 + a^2}\,dx = \frac{b}{2}\ln|x^2 + a^2| + \frac{c}{a}\arctan\left(\frac{x}{a}\right) + C,\ a \neq 0$

26. $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{(a-b)}(\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$

27. $\int \frac{cx + d}{(x-a)(x-b)}\,dx = \frac{1}{(a-b)}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C, \quad a \neq b$

VI. Integrands involving $\sqrt{a^2+x^2}, \sqrt{a^2-x^2}, \sqrt{x^2-a^2}, a > 0$

28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$

29. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}| + C$

30. $\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$

31. $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}\,dx\right) + C$