

# Polytechnic University

MA 1122

MIDTERM

APRIL 17, 2006

Print Name:

Signature:

ID #:

Instructor/Section:

**Directions:** You have **90 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last page contains formulas that you might find useful. You may tear that page out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	14	
2	12	
3	10	
4	10	
5	10	
6	10	
7	12	
8	12	
9	10	
Total	100	

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- (1) (Page 355, Problems 10–14) Evaluate each improper integral or show that it diverges. Do not use any comparison test. Show all your work.

(a) 
$$\int_1^{\infty} \frac{x}{(1+x^2)^2} dx$$

(b) 
$$\int_0^{\ln(3)} \frac{e^x}{\sqrt{e^x - 1}} dx$$

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- (2) (Page 359, Problems 18 and 24) Use a comparison test to determine whether each of the following converges or diverges. If it is convergent, find an upper bound. Show all your work.

(a)  $\int_0^{\infty} \frac{\sin^2(x)}{x^2 + 1} dx$

(b)  $\int_2^{\infty} \frac{1}{\sqrt{x+2}-1} dx$

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- (3) (Page 373, Problem 8) Find the area of the region bounded by  $y = x + 6$ ,  $y = x^3$  and  $2y + x = 0$ , where the points of intersection of the three graphs pairwise are  $(-4, 2)$ ,  $(0, 0)$  and  $(2, 8)$ . Show all your work.

Hint: Sketch the graphs of the given functions.

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- (4) (Page 380, Problems 9 and 30) **Set up**, but do not evaluate, the volume of the solid generated by revolving the region bounded by the line  $y = 4(x - 3)$  and the parabola  $y = 4(x - 3)^2$  about the line  $y = 5$ .

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(5) (Concepts) Decide whether each of the following statements is TRUE or FALSE. You do not have to explain.

(a) If  $f$  is an even function and  $\int_0^{\infty} f(x) dx$  converges, then  $\int_{-\infty}^{\infty} f(x) dx$  converges.

(b) If a sequence is bounded, then it is convergent.

(c) You can tell if a sequence converges by looking at the first 10000 terms.

(d) The improper integral  $\int_1^{\infty} \frac{1}{x^2 \ln(x+1)} dx$  is convergent.

(e) If  $c$  is a constant, then the geometric series  $\sum_2^{\infty} \left(\frac{1}{1+c}\right)^n$  is convergent.

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(6) (Page 380, Problem 12) Find the arc length of the graph of the function

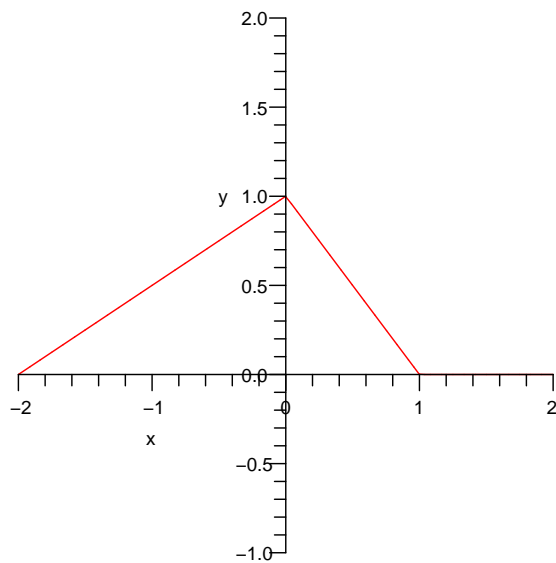
$$f(x) = x^2 - \frac{\ln(x)}{8}$$

from  $x = 1$  to  $x = e$ . Show all your work.

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- (7) (Page 397, Problem 2 and 10) A region composed of a triangular region as shown below, has a density  $\delta(x) = 2x + 5$  g/cm<sup>2</sup>.



- (a) Find the total mass of the region. Show all your work.
- (b) Find the  $x$ -coordinate of the center of mass of the region. Show all your work.

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- (8) (Page 449, Problem 28) A ball is dropped from a height of 100 feet. Each time it hits the floor, it rebounds to two-third of its previous height. Find the total distance it travels if it bounces forever. Show all your work.

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- (9) (Page 449, Problems 19 and 20) Find the **exact** sum of each of the following expressions if possible. You may want to list a few terms. Show all your work.

(a) 
$$\sum_{n=1}^{10} \left(\frac{3}{4}\right)^n$$

(b) 
$$\sum_{k=1}^{\infty} \left(\frac{e}{\pi}\right)^{k+1}$$

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### Useful formulas

- *Physics formulas:*

The *acceleration* due to gravity,  $g$ :  $g = 9.8\text{m/sec}^2$ , or  $g = 32\text{ft/sec}^2$ .

Mass density of water =  $1000\text{ kg/m}^3$ ,      Weight density of water =  $62.4\text{ lbs/ft}^3$ .

The center of mass,  $\bar{x}$ , of an object lying on the  $x$ -axis between  $x = a$  and  $x = b$ ,

with mass density  $\delta(x)$  is given by  $\bar{x} = \frac{\int_a^b x\delta(x) dx}{\text{total mass}}$

The center of mass,  $\bar{x}$ , of  $n$  **discrete** masses  $m_i$  lying along the  $x$ -axis, each located

at  $x_i$  is given by  $\bar{x} = \frac{\sum_{i=1}^n x_i m_i}{\sum_{i=1}^n m_i}$

Arc length of a curve  $y = f(x)$  from  $x = a$  to  $x = b$ :  $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$

- *Integration by Parts:*

$$\int u dv = uv - \int v du \quad \text{or} \quad \int uv' dx = uv - \int vu' dx$$

- *Numerical Approximations:*

$$\text{TRAP}(n) = \frac{\text{LEFT}(n) + \text{RIGHT}(n)}{2}; \quad \text{SIMP}(n) = \frac{2\text{MID}(n) + \text{TRAP}(n)}{3}$$

- *Useful Integrals for Comparison:*

$$\int_1^{\infty} \frac{1}{x^p} dx \text{ converges for } p > 1 \text{ and diverges for } p \leq 1.$$

$$\int_0^1 \frac{1}{x^p} dx \text{ converges for } p < 1 \text{ and diverges for } p \geq 1.$$

$$\int_0^{\infty} e^{-ax} dx \text{ converges for } a > 0.$$

- *Finite Geometric Sum:*

$$a + ax + ax^2 + \cdots + ax^{n-1} = \frac{a(1 - x^n)}{1 - x}$$

- *Infinite Geometric Series:*

$$a + ax + ax^2 + \cdots = \frac{a}{1 - x} \quad \text{for } |x| < 1$$

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• Differentiation formulas

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln x ) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

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Here  $a, b, c, d$  are constants.

## A Short Table of Indefinite Integrals

### I. Basic Functions

$$\begin{array}{l} 1. \int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad (n \neq -1) \\ 2. \int \frac{1}{x} dx = \ln|x| + C \\ 3. \int a^x dx = \frac{1}{\ln a}a^x + C \\ 4. \int \ln x dx = x \ln x - x + C \end{array} \quad \left\| \begin{array}{l} 5. \int \sin ax dx = -\frac{1}{a} \cos ax + C \\ 6. \int \cos ax dx = \frac{1}{a} \sin ax + C \\ 7. \int \tan ax dx = -\frac{1}{a} \ln|\cos ax| + C \end{array} \right.$$

### II. Products of $e^x$ , $\cos x$ , and $\sin x$

$$\begin{array}{l} 8. \int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C \\ 9. \int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C \\ 10. \int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 11. \int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b \\ 12. \int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b \end{array}$$

### III. Product of Polynomial $p(x)$ with $\ln x, e^x$ , $\cos x$ , and $\sin x$

$$\begin{array}{l} 13. \int x^n \ln x dx = \frac{1}{n+1}x^{n+1} \ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, x > 0 \\ 14. \int p(x)e^{ax} dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \dots + C \\ \quad (+ - + - + - + \dots) \text{ (signs alternate)} \\ 15. \int p(x) \sin ax dx = -\frac{1}{a}p(x) \cos(ax) + \frac{1}{a^2}p'(x) \sin(ax) + \frac{1}{a^3}p''(x) \cos(ax) - \dots + C \\ \quad (- + + - - + + - - \dots) \text{ (signs alternate in pairs)} \\ 16. \int p(x) \cos ax dx = \frac{1}{a}p(x) \sin(ax) + \frac{1}{a^2}p'(x) \cos(ax) - \frac{1}{a^3}p''(x) \sin(ax) - \dots + C \\ \quad (+ + - - + + - - \dots) \text{ (signs alternate in pairs)} \end{array}$$

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#### IV. Integer Powers of $\sin x$ and $\cos x$

$$17. \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \quad n \text{ positive}$$

$$18. \int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx, \quad n \text{ positive}$$

$$19. \int \frac{1}{\sin^m x} \, dx = -\frac{1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$$

$$20. \int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$$

$$21. \int \frac{1}{\cos^m x} \, dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$$

$$22. \int \frac{1}{\cos x} \, dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$$

$$23. \int \sin^m x \cos^n x \, dx :$$

If  $n$  is odd, let  $w = \sin x$ .

If both  $m$  and  $n$  are even and non-negative, convert all to  $\sin x$  or all to  $\cos x$  (using  $\sin^2 x + \cos^2 x = 1$ ), and use IV-17 or IV-18.

If  $m$  and  $n$  are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.

The case in which both  $m$  and  $n$  are even and negative is omitted.

#### V. Quadratic in the Denominator

$$24. \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left( \frac{x}{a} \right) + C, \quad a \neq 0$$

$$25. \int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + \frac{c}{a} \arctan \left( \frac{x}{a} \right) + C, \quad a \neq 0$$

$$26. \int \frac{1}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} (\ln |x-a| - \ln |x-b|) + C, \quad a \neq b$$

$$27. \int \frac{cx + d}{(x-a)(x-b)} \, dx = \frac{1}{(a-b)} [(ac + d) \ln |x-a| - (bc + d) \ln |x-b|] + C, \quad a \neq b$$

#### VI. Integrands involving $\sqrt{a^2 + x^2}$ , $\sqrt{a^2 - x^2}$ , $\sqrt{x^2 - a^2}$ , $a > 0$

$$28. \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \left( \frac{x}{a} \right) + C$$

$$29. \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$30. \int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left( x\sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C$$

$$31. \int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left( x\sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C$$