Polytechnic University

MA 1132 — Worksheet IV
Instructor: Manocha

Due: Wednesday, March 7, 2007
You must show all of your work for problem 1.

(1) (Page 485, Problem 30) Use a fourth degree Taylor approximation for e^h, for h near 0, to evaluate the following limits. Would your answer be different if you used a Taylor polynomial of higher degree?

(a) $\lim_{h\to 0} \dfrac{e^h - 1 - h}{h^2}$

(b) $\lim_{h\to 0} \dfrac{e^h - 1 - h - \frac{h^2}{2}}{h^3}$

(2) Suppose g is a function which has a continuous derivatives, and that $g(5) = 4$, $g'(5) = -3$, $g''(5) = 2$, $g'''(5) = -4$. What is the Taylor polynomial of degree 2 for g near 5? (Hint: You will need to convert any factorials into integers.) Circle the correct choice. You do not need to show work.

(a) $P_2(x) = 4 - 3(x-5) + (x-5)^2$
(b) $P_2(x) = 4 - 3(x-5) + 2(x-5)^2$
(c) $P_2(x) = 1 - 3(x-5) + (x-5)^2$
(d) $P_2(x) = 1 - 2(x-5) + 3(x-5)^2$
(e) $P_2(x) = 4 - 3(x-5) - 2(x-5)^2$

(3) Find the third-degree Taylor polynomial for $f(x) = x^3 + 2x^2 - 7x + 9$ about $x = 0$. Circle the correct choice. You do not need to show work.

(a) $P_3(x) = x^3 + 2x^2 + 3x + 4$
(b) $P_3(x) = x^3 + x^2 + x + 1$
(c) $P_3(x) = x^3 - 2x^2 + 7x - 9$
(d) $P_3(x) = x^3 + 2x^2 - 7x + 9$
(e) $P_3(x) = x^3 - 2x^2 + 3x + 4$

(4) By recognizing the series below as a Taylor series evaluated at a particular value of x, find the sum of the convergent series.

$$1 + \frac{4}{1!} + \frac{16}{2!} + \frac{64}{3!} + \ldots + \frac{4^n}{n!} + \ldots$$

Circle the correct choice. You do not need to show work.

(a) 14.6
(b) 24.6
(c) 34.6
(d) 44.6
(e) 54.6

(5) By recognizing the series below as a Taylor series evaluated at a particular value of x, find the sum of the convergent series.

$$1 + \frac{1}{6} + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \cdots + \left(\frac{1}{6}\right)^n + \ldots$$

Circle the correct choice. You do not need to show work.

(a) 0.2
(b) 1.2
(c) 2.2
(d) 3.2
(e) 4.2

(6) By looking at the Taylor series, put the following functions in increasing order for small positive x.

(a) $\frac{1}{1-x^2}$
(b) $1 + \sin(x)$
(c) $\cos(x)$

Circle the correct choice. You do not need to show work.

(i) $c < a < b$
(ii) $a < c < b$
(iii) $a < b < c$
(iv) $c < b < a$
(v) $b < a < c$

(7) Find the fourth term of the Taylor series about 0 for the function $\frac{z^5}{e^{z^5}}$. Circle the correct choice. You do not need to show work.

(a) $\frac{z^6}{6}$
(b) $\frac{z^{20}}{6}$
(c) $\frac{-z^{20}}{20}$
(d) $\frac{z^{20}}{20}$
(e) $\frac{-z^{20}}{6}$