1. Find \( x \) and \( B \) so that the matrix
\[
B = \begin{pmatrix}
x & x + 2 \\
2x - 3 & x + 1
\end{pmatrix}.
\]
(a) is symmetric in \( \mathbb{R} \),
(b) is skew-symmetric in \( \mathbb{R} \),
(c) is skew-symmetric in \( \mathbb{Z}_2 \).

2. Small proofs.
   (a) Show that if \( A \) is a square matrix, then \( AA^T \) is symmetric.
   (b) Show that if \( A \) and \( B \) are orthogonal, so is \( AB \).
   (c) In class we showed that \( A + A^T \) is symmetric and \( A - A^T \) is skew-symmetric for any square matrix \( A \). Use this to show that every square matrix \( B \) can be written as the sum of a symmetric and a skew-symmetric matrix.

3. Let
\[
A = \begin{pmatrix}
a & -3 & 2 \\
0 & 2 & 5 \\
0 & 0 & 3
\end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix}
b & 5 & 7 \\
0 & 2b & 2 \\
10 & 0 & 3
\end{pmatrix}
\]
Find numbers \( a \) and \( b \) for which the following system of equations holds:
\[
\begin{cases}
det(2B^{-1}AB) = 36 \\
tr(3A - aB) = 13.
\end{cases}
\]

4. Calculate the determinant
\[
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 \\
r & 1 & 1 & 1 & 1 \\
r & r & 1 & 1 & 1 \\
r & r & r & 1 & 1 \\
r & r & r & r & 1
\end{vmatrix},
\]
then determine the value(s) of $r$ for which the matrix is singular.

**Hint:** You may want to use elementary row operations. How does the value of the determinant change during the process?

5. Let

$$M = \begin{pmatrix}
1 & k & k & 1 & k \\
1 & k & k & 3 & 4 \\
1 & 1 & k & -1 & 2 \\
0 & 0 & 0 & 1 & k \\
0 & 0 & 0 & k & 2k \\
0 & 0 & 0 & 4 & 5
\end{pmatrix}. $$

Write $M$ as a square block matrix and find the determinant of $M$. For what values of $k$ is the matrix $M$ singular.

6. For what value of $k$ does the matrix

$$A = \begin{pmatrix}
1 & 0 & k \\
2 & -1 & 3 \\
4 & 1 & -3
\end{pmatrix} $$

have an inverse?

(a) Find $\text{adj}(A)$.

(b) For what values of $k$ does $A^{-1}$ exist? Find $A^{-1}$. 