

Math 2112
Worksheet II

Exercise 1.

- (1) Find all the vectors \vec{v} in 2 dimensions having $\|\vec{v}\| = 12$ and its \vec{j} -component is $2\vec{j}$.
- (2) Find a unit vector from the point $P = (-1, 2)$ and toward the point $Q = (2, -1)$

Exercise 2. Consider the vectors: $\vec{a} = \vec{i} - 2\vec{k}$, $\vec{b} = 4\vec{i} - 4\vec{j} + 12\vec{k}$, $\vec{c} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{d} = -2\vec{i} + 4\vec{k}$, $\vec{e} = -2\vec{i} + \vec{k}$, $\vec{f} = \vec{i} - \vec{j} + 3\vec{k}$.

- (1) Which of the above vectors are parallel?
- (2) Which of the above vectors are perpendicular to each other?

Exercise 3. Find a vector perpendicular to the plane that passes through the points $(3, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 5)$. Use this vector to write down an equation for this plane.

Exercise 4.

- (1) Show that the planes $x + (y - 2) - (z + 5) = 0$ and $2z - 2y - 2(x - 4) = 7$ are parallel.
- (2) Find the angle between the planes $2(x + 7) - y - (z - 2) = 0$ and $x - (y - 2) + 5 = 0$.

Exercise 5.

- (1) If $\vec{v} = 2\vec{i} - 3\vec{j} + 6\vec{k}$ and $\vec{w} = \vec{i} + 2\vec{j} - \vec{k}$, find $\vec{v} \times \vec{w}$ and $\vec{w} \times \vec{v}$. What is the relation between the two answers.
- (2) If $\vec{a} \times \vec{b} = \vec{i} + 2\vec{j} - 2\vec{k}$ and $\vec{a} \cdot \vec{b} = -3$, find $\tan \theta$, where θ is the angle between \vec{a} and \vec{b} .
- (3) Find a vector parallel to the line of intersection of the two planes $x + (y - 2) - (z + 4) = 0$ and $(x + 7) - (y + 1) - z = 2$.

Exercise 6. Let $g(x, y) = 2x \sin(y) + y^3 e^x$ and $h(x, y) = x^2 \cos(y) + ky^2 e^x$.

- (1) Find k such that $g_y(x, y) = h_x(x, y)$.
- (2) When this condition is true, one can find a function $F(x, y)$ such that $F_x(x, y) = g(x, y)$ and $F_y(x, y) = h(x, y)$. Find one such $F(x, y)$. Are there any other?

Exercise 7. A few of the level curves for a function $f(x, y)$ are indicated in the table below.

$c = -20$	$x^2 + y^2 = 36$
$c = -9$	$x^2 + y^2 = 25$
$c = 0$	$x^2 + y^2 = 16$
$c = 7$	$x^2 + y^2 = 9$
$c = 15$	$x^2 + y^2 = 1$

For each of the following quantities, decide whether it is positive, negative or zero. Explain your answer. (Hint: Draw the contour diagram.)

- (1) f_x at the point $(0, 1)$.
- (2) f_y at the point $(0, -4)$.
- (3) $\nabla f \cdot (\vec{i} + \vec{j})$ at the point $(3, 0)$.
- (4) $\nabla f \cdot (-\vec{i} - \vec{j})$ at the point $(3, 4)$.