Math 2112

Worksheet III

Exercise 1. A function is given by $f(x, y) := (x-1)^2(y-2)$.

(1) Find the equation of the tangent plane at the origin.
(2) This tangent plane is not parallel to the xy-plane. Why is that clear from your answer to part (1)?
(3) Find the set of all points (x_0, y_0) at which the tangent plane to the graph of f at $(x_0, y_0, f(x_0, y_0))$ is parallel to the xy-plane.

Exercise 2.

(1) Find the equation of the plane tangent to the surface $z = x^2 + y^2$ at the point $x = 3, y = 4$.
(2) Find the components of the normal vector to that plane.
(3) Find two points in the plane and find a vector lying in the plane.
(4) Show that the vector in the plane is indeed perpendicular to the normal vector.
(5) Find the distance from the origin to the plane.

Exercise 3. (Midterm 2/14/2002) The temperature T on a disk of radius 10 centered at the origin is given by $T(x, y) = 100 - x^2 - y^2$.

(1) Sketch the isothermal (constant temperature) curves for $T = 100, T = 75$, and $T = 50$.
(2) Suppose a heat seeking bug is placed anywhere on the disk. In which direction should it move to increase its temperature the fastest?
(3) How is this direction related to the level curve through that point?

Exercise 4.

(1) If $H(r, s) = f(r^2 + s^2)$, calculate $H_s(-1, 2)$ if $f'(5) = -3$.
(2) Let $w = f(ut, t^2)$ be the function of u and t obtained by substituting $x = ut$ and $y = t^2$ in the expression for $f(x, y)$, an unspecified function. Find w_t and w_{tu} at $(t, u) = (2, 1)$, given that $f_x(2, 4) = f_y(2, 4) = 5$ and $f_{xx}(2, 4) = 1,\ f_{xy}(2, 4) = -2$, and $f_{yy}(2, 4) = -1$.

Exercise 5. At time $t = 0$ a particle leaves the point $(1, 1, 1)$ at a speed of 10 m/s. Its path lies along the outward normal to the surface with equation $x^2+2y^2+3z^2 = 6$. At what time does it cross the sphere $x^2 + y^2 + z^2 = 10$?

Exercise 6.

(1) Find the quadratic function that best approximates the function $f(x, y) := \sin(x)\sin(y)$ around $(\pi/2, \pi/2)$.
(2) Find the quadratic function that best approximates the function $g(x, y) := \exp(-x^2 - y^2)$ around the origin.