

Math 2112
Worksheet IV

Exercise 1.

- (1) Find and classify the critical points of the function $f(x, y) = y^2 - \cos x$.
- (2) The origin is a critical point for the function $f(x, y) = \sin(xy)$. What sort of critical point is it? Explain.

Exercise 2. Suppose $f(x, y) = Ax^3 + Bxy + Cy^2$, where A, B and C are constants. For what values of A, B, C does f have a critical value of 15 at the point $(-2, 1)$? Classify the critical point as a local maximum, local minimum or a saddle point.

Exercise 3. Find the global maximum and minimum of the given function on the square $-1 \leq x \leq 1, -1 \leq y \leq 1$, and say whether it occurs on the boundary.

- (1) $f(x, y) = xy$
- (2) $f(x, y) = \exp(-x^2 - y^2)$

Exercise 4. Use the Lagrange multipliers method to solve the following problems:

- (1) Find the minimum distance from the origin to the plane $z = x + y - 2$.
- (2) Find the minimum distance from the point $(\sqrt{8}, -\sqrt{8}, 2)$ to the surface given by the equation $z^2 = x^2 + y^2$.
- (3) Find the maximum of the function $f(x, y, z) := x^2y^2z^2$ subject to the constraint $x^2 + y^2 + z^2 = c^2$ where $c \neq 0$ is a constant.

Exercise 5. (Final, 3/17/2004) Consider the function

$$f(x, y) = A(xy + B)(x + y)$$

where A and B are constants, $A \neq 0$. Determine whether each of the following statements is TRUE or FALSE. Explain.

- (1) f has, at least, two critical points for any value of the constant B .
- (2) For any $B \neq 0$ and $A > 0$, f has two saddle points.
- (3) If $A < 0$ and $B > 0$, f has a local minimum at $(\sqrt{B}, -\sqrt{B})$, and a local maximum at $(-\sqrt{B}, \sqrt{B})$.
- (4) If $B = 0$, f has a saddle point at the origin.
- (5) For any B , the global maximum and minimum of f on $0 \leq x \leq 1, 0 \leq y \leq 1$ occur on the boundary of the square.

Exercise 6.

- (1) A particle starts at the point $P = (1, -1, 2)$ and moves along a straight line toward $Q = (0, 2, -1)$ at a speed of 3 cm/sec. Find the particle's velocity and position vectors.
- (2) A particle starts at the point $P = (1, -1, 2)$ and moves clockwise along the circle $x^2 + y^2 = 2, z = 2$ at a speed of 3 cm/sec. Find the particle's velocity and position vectors.