Algebra and Calculus in MA2132

You will see many techniques for solving differential equations in the course. Each technique really involves little more than using pre-calculus algebra, calculus, and some linear algebra in the right order. The "hard" part is learning and remembering the sequence of steps in each method, and the "easy" part should be the calculus and algebra involved in each step. Make sure you always check your algebra, it is very easy to turn a straightforward question into a very difficult question by making a careless algebra error early on. Here are some topics from pre-requisite courses which you should be comfortable with.

(1) **Functions and independent variables:** The use of different letters can cause confusion, particularly x. You should always establish which letter(s) are functions, and which is the variable before you attempt to solve any problem. Probably the easiest "rule of thumb" is that you should only have derivatives of functions, and there is probably only one variable which you may have to name yourself. For example in the equation $x' = tx$, t must be the variable and x is a function of t. In the equation $y' = y^2$ you can choose any letter for the variable, other than y which denotes the function. In the system $y' = x, x' = y$ both x and y are functions, and usually we would use t for the variable.

(2) **Logarithms:** If $e^y = e^x + C$ then $y = \ln(e^x + C)$. Remember that $\ln(a+b) \neq \ln(a) + \ln(b)$.

(3) **Exponentials:** Make sure you understand the rules for exponents, this is particularly important when finding integrating factors. For example $e^{3\ln(x)} = x^3$ and $e^{-5\ln(x)} = 1/x^5$.

(4) **Factoring Polynomials:** You should know how to factor polynomials and find their roots, this is needed for characteristic equations and also for finding eigenvalues. For quadratic polynomials there is the quadratic formula, but for higher order polynomials the formulas are either very, very complicated or non-existent. Instead you should look for "obvious" roots ($0, 1, 2-1, -2$ etc.) and divide out the corresponding factors using long division, and then repeat the process if necessary until you find all the roots. For example, -1 is a root of $r^3 + 1$ because $(-1)^3 + 1 = 0$ and so $r + 1$ is a factor of $r^3 + 1$. Long division gives $r^3 + 1 = (r + 1)(r^2 - r + 1)$ and then you find the roots of $r^2 - r + 1$ using the quadratic formula. A polynomial will have as many roots as its degree, though some may be repeated roots.

(5) **Integrals:** Understand how to do integrals by substitution, by parts, and using partial fractions. You should know how to find anti-derivatives of functions such as $\tan(x)$, xe^x and $1/(x^2 - x)$ without using a calculator.

Do not forget the constant(s) of integration. Sometimes we assume for simplicity that constants are 0 or 1 but this is ONLY when we are looking for ONE function (for example, finding an integrating factor or solving the Variation of Parameters equations). A general solution always contains constant(s).

(6) **Matrices:** You should know how (and when) to multiply matrices and vectors and the dimensions of the solution. You should be able to calculate the determinant of any square matrix using the row operations/triangular form method and also by the cofactor/expansion formulas.

A large part of the second half of the course involves finding eigenvalues (solve the polynomial equation $\det(A - \lambda I) = 0$ to find λ) and eigenvectors corresponding the eigenvalue λ (solve the vector equation $(A - \lambda I)\mathbf{v} = \mathbf{0}$ to find $\mathbf{v}$) so you need to understand these concepts from linear algebra.