Polytechnic University

MA 2132 FINAL JANUARY 16TH, 2007

Print Name:

Signature:

ID #:

Instructor/Section: Andy Tsang

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator, **but you must show your work for integrals and derivatives**.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID, and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Your signature:

(1) (Worksheet 5, Problem 1) Consider the 3rd order linear differential equation:

$$y''' + y' = 0$$

(a) Find three linearly independent solutions of the differential equation. You must show that they are linearly independent.

(b) Write the 3rd order linear differential equation as a 3-dimensional system of 1st order linear differential equations. Show all your work.

(2) (Worksheet 4, Problem 4) Find the solution of the initial value problem to the 2-dimensional system of 1st order linear differential equations:

$$x' = 3x - y, \quad x(0) = 1$$
$$y' = x + 5y, \quad y(0) = 2$$

Show all your work.

(3) (a) (Worksheet 4, Problem 3) Find the general solution of the of the 2-dimensional system of 1st order linear differential equations:

$$x' = -x - 2y$$
$$y' = x + y$$

Show all your work.

(b) (Worksheet 5, Problem 4) Given the following 2x2 matrix:

$$A = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$$

Find e^{At} in the form of a 2x2 matrix. Show all your work.

(4) (Worksheet 5, Problem 3) Find the general solution of the 2-dimensional system of 1st order linear differential equations:

$$x' = x + 3y + e^t$$
$$y' = x - y$$

Show all your work.

(5) (Worksheet 5, Problem 5) Consider the initial value problem:

$$y' = \frac{y}{t+1} + \frac{1}{(t+1)^2}, \quad y(0) = 2$$

(a) Use the Euler method with step size h=0.1 to approximate y(0.3) up to four decimal places. Show all of your work.

(b) Find the solution of the initial value problem and evaluate y(0.3) up to four decimal places. Show all of your work.

FORMULA SHEET

(a) **Integration By Parts:** $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$.

(b) **Partial Fractions Integral:** If $c \neq d$ then

$$\int \frac{ax+b}{(x-c)(x-d)}dx = \frac{1}{c-d}\big[(ac+b)\ln|x-c| - (ad+b)\ln|x-d|\big] + K.$$

(c) **The Logistic Equation:** $P' = r_0(1 - \frac{P}{K})P$ has the implicit general solution

$$\frac{P}{K-P} = \frac{P_0}{K-P_0}e^{r_0 t}$$

(d) **Variation of Parameters:** If y_1 and y_2 are linearly independent solutions of the equation $y'' + p(t)y' + q(t)y = 0$, then $y_p = v_1y_1 + v_2y_2$ is a particular solution of the equation $y'' + p(t)y' + q(t)y = f(t)$, where v_1 and v_2 satisfy the VOP equations

$$\begin{aligned} v_1'y_1 + v_2'y_2 &= 0 \\ v_1'y_1' + v_2'y_2' &= f(t). \end{aligned}$$

(e) **Matrix Exponential:** If A is a square matrix and t is a variable then

$$e^{tA} = I + tA + \frac{t^2}{2!}A^2 + \frac{t^3}{3!}A^3 + \cdots$$

(f) **Generalized Eigenvector:** If λ is a an eigenvalue for a square matrix A, then $\mathbf{v}$ is a corresponding generalized eigenvector if $(A - \lambda I)^d\mathbf{v} = \mathbf{0}$ for some positive integer d.

(g) **Variation of Parameters for First Order Linear Systems:** If $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}$ are linearly independent vector solutions of the n-dimensional homogeneous linear system $\mathbf{x}' = A\mathbf{x}$, then

$$\mathbf{x_p} = Y\int Y^{-1}\mathbf{f}dt$$

is a particular solution of the system $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where Y is the $n \times n$ matrix

$$\begin{bmatrix}\mathbf{x_1} \ \mathbf{x_2} \ \ldots \ \mathbf{x_n}\end{bmatrix}.$$