Polytechnic University

MA 2132 MIDTERM April 15th, 2003

Print Name:

Signature:

ID #:

Lecture Instructor: Gonye Zauderer

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator, **but you must show your work for integrals and derivatives**. There are formulas on the last page of the exam which you may detach.

Problem	Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

(1) (20 points) Consider the initial value problem

$$t^2 z' = tz + z^2, \qquad z(1) = 1$$

(a) Find an **explicit** formula for $z(t)$.

(b) What is the interval of existence for this solution?

(2) (20 points) Find the general solution of the first order differential equation

$$\frac{dy}{dx} = -\frac{e^x + ye^{xy}}{e^y + xe^{xy}}$$

(3) (20 points) Solve the initial value problem for the second order ODE

$$y'' + 2y' + y = e^t + e^{-t}, \qquad y(0) = 1, \quad y'(0) = -1$$

(4) (20 points) Consider the homogenous, second order linear ODE

$$t^2y'' - 3ty' + 3y = 0, \text{ for } t > 0$$

(a) Use the substitution $y = t^r$ to find two linearly independent solutions y_1 and y_2 of the equation. Use the Wronskian to verify that y_1 and y_2 are linearly independent.

(b) Find a particular solution for the second order ODE

$$t^2y'' - 3ty' + 3y = t^3$$

(5) (20 points) A rabbit population $P(t)$ grows according to the logistic equation

$$P' = \frac{P}{12} - \frac{P^2}{1800}$$

where t is measured in months.

(a) Find the *equilibrium* solutions for the equation and determine whether they are stable or unstable.

(b) If initially there are 50 rabbits in the population, after how many months will there be 100 rabbits?

Formula Sheet

(1) **Integration By Parts:** $\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$

(2) **Partial Fractions Integral:** If $c \neq d$ then

$$\int \frac{ax+b}{(x-c)(x-d)}\,dx = \frac{1}{c-d}((ac+b)\ln|x-c| - (ad+b)\ln|x-d|) + K$$

(3) **The Logistic Equation:** $P' = r_0(1-P/K)P$ has the implicit general solution

$$\frac{P}{K-P} = \frac{P_0}{K-P_0}e^{r_0 t}$$

(4) **Variation of Parameters:** If y_1 and y_2 are linearly independent solutions of the equation $y'' + p(t)y' + q(t)y = 0$, then $y_p = v_1y_1 + v_2y_2$ is a particular solution of the equation $y'' + p(t)y' + q(t)y = f(t)$, where v_1 and v_2 satisfy the VOP equations

$$\begin{aligned} v_1'y_1 + v_2'y_2 &= 0 \\ v_1'y_1' + v_2'y_2' &= f(t). \end{aligned}$$