Polytechnic University

MA 2132 MIDTERM FEBRUARY 13, 2006

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator, **but you must show your work for integrals and derivatives**. There are formulas on the last page of the exam which you may detach.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	17	
2	10	
3	20	
4	15	
5	18	
6	20	
Total	100	

(1) Consider the IVP

$$x' = -12e^{x}t^{3}, \quad x(-2) = 0.$$

(a) Find the solution of the IVP in explicit form.

(b) Find the interval of existence for the solution.

(2) (a) Use an appropriate substitution to transform the following non-linear differential equation to a linear first-order differential equation. (You do not need to solve!)

$$(1 + 2t)x' = 2x + (4 + 8t)x^5$$

(b) When solving the first-order linear differential equation

$$2xy' + y = 2x^2 e^x$$

we write the equation in the form $\frac{d}{dx}\big(yf(x)\big) = g(x)$. Find $f(x)$ and $g(x)$. (You do not need to solve the equation.)

$f(x) =$____________, and $g(x)=$____________.

(3) Consider the ODE

$$(x^4 + y)dx + (xy^2 + x \ln x)dy = 0, \qquad x > 0.$$

(a) Show that the ODE is **not** an exact differential equation, but it becomes exact if you multiply by the integrating factor $\frac{1}{x}$.

(b) Find the general solution of the ODE in implicit form.

(c) Find the implicit solution of the IVP

$$(x^4 + y)dx + (xy^2 + x \ln x)dy = 0, \qquad y(1) = \frac{3}{2}.$$

(4) Throughout this problem we consider the autonomous differential equation

$$y' = y^3 - 2y^2.$$

State whether each of the following statements is TRUE or FALSE. You do not need to explain your answers.

(a) $y = 0$ is an (asymptotically) stable equilibrium solution of the equation.

(b) $y = 2$ is an (asymptotically) stable equilibrium solution of the equation.

(c) If $y(t)$ is the particular solution of the equation with initial condition $y(0) = 1$ then $y(0) < y(2)$.

(d) If $y(t)$ is the particular solution of the equation with initial condition $y(0) = 2$ then $\lim_{t \to \infty} y(t) = 2$.

(e) If $y(t)$ is the particular solution of the equation with initial condition $y(2) = 0$ then $\lim_{t \to \infty} y(t) = 2$.

(5) Find the general solution of the differential equation

$$y'' - 2y' - 15y = 2e^{-3x}.$$

(6) (a) Find two linearly independent solutions, y_1 and y_2, of the second order, linear homogeneous differential equation

$$t^2y'' - 6ty' + 12y = 0, \qquad t > 0.$$

(b) Use the Wronksian to verify that y_1 and y_2 are linearly independent.

(c) (Continued from the previous page.) Use variation of parameters to find the general solution of the non-homogeneous equation

$$t^2 y'' - 6ty' + 12y = \frac{t^5}{1+t^2}, \qquad t > 0.$$

FORMULA SHEET

(1) **Integration By Parts:** $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$.

(2) **Partial Fractions Integral:** If $c \neq d$ then

$$\int \frac{ax+b}{(x-c)(x-d)}dx = \frac{1}{c-d}\big[(ac+b)\ln|x-c| - (ad+b)\ln|x-d|\big] + K.$$

(3) **The Logistic Equation:** $P' = r_0(1-P/K)P$ has the implicit general solution

$$\frac{P}{K-P} = \frac{P_0}{K-P_0}e^{r_0 t}.$$

(4) **For linear homogeneous d.e. with constant coefficients:** $y'' + by' + cy = 0$.

- If $b^2 - 4c > 0$, then r_1 and r_2 are two distinct solutions of the characteristic equation and

$$y = C_1 e^{r_1 t} + C_2 e^{r_2 t},$$

where C_1 and C_2 are constants.

- If $b^2 - 4c = 0$, then there is only one solution of the characteristic equation, $r = -b/2$, and

$$y = C_1 t e^{rt} + C_2 e^{rt}.$$

- If $b^2 - 4c < 0$, then the solutions of the characteristic equation are of the form $r = \alpha \pm \beta i$ and

$$y = C_1 e^{\alpha t}\cos(\beta t) + C_2 e^{\alpha t}\sin(\beta t).$$

(5) **For linear non-homogeneous d.e. with constant coefficients:**

If $f(x)$ is	then try $y_p(x)$ in the form of
polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$	polynomial of same degree $A_n x^n + A_{n-1}x^{n-1} + \cdots + A_0$
be^{kx}	Be^{kx}
$b\sin(ax)$ or $b\cos(ax)$	$B\sin(ax) + C\cos(ax)$

(6) **Variation of Parameters:** If y_1 and y_2 are linearly independent solutions of the equation $y'' + p(t)y' + q(t)y = 0$, then $y_p = v_1 y_1 + v_2 y_2$ is a particular solution of the equation $y'' + p(t)y' + q(t)y = f(t)$, where v_1 and v_2 satisfy the VOP equations

$$\begin{aligned} v_1' y_1 + v_2' y_2 &= 0 \\ v_1' y_1' + v_2' y_2' &= f(t). \end{aligned}$$