Polytechnic University

MA 2132 MIDTERM APRIL 7, 2006

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	12	
2	15	
3	15	
4	15	
5	18	
6	15	
7	10	
Total	100	

YOUR SIGNATURE:

(1) In each of the following parts, circle the **ALL** correct answers which describe the type of the given first order differential equation. You do not need to explain.

(a) $(x^2 + 4)y' + 6x = 3xy$

(i) Separable.

(ii) Autonomous.

(iii) Linear.

(iv) Bernoulli, but not linear.

(v) None of the above are correct.

(b) $y \cos(x) y' = e^x (y^2 + 1)$

(i) Separable.

(ii) Autonomous.

(iii) Linear.

(iv) Bernoulli, but not linear.

(v) None of the above are correct.

(c) $y' = 2xy + 3e^x y^2$

(i) Separable.

(ii) Autonomous.

(iii) Linear.

(iv) Bernoulli, but not linear.

(v) None of the above are correct.

(d) $\dfrac{dx}{dt} = kx(x - M)$ (Here k and M are constants.)

(i) Separable.

(ii) Autonomous.

(iii) Linear.

(iv) Bernoulli, but not linear.

(v) None of the above are correct.

YOUR SIGNATURE: ____

(2) For each of the following parts (a)- (c) below, circle the **ONE** alternative that best completes the sentence. You do not need to explain.

(a) The equilibrium solution $y = -2$ of the differential equation $\dfrac{dy}{dt} = y^3 - 4y$ is

(i) asymptotically stable.

(ii) unstable.

(b) If $y(t)$ is the solution of the initial value problem $y' = y^3 - 4y,\ y(2) = 1$, then

(i) $\lim_{t\to\infty} y(t) = -\infty$.

(ii) $\lim_{t\to\infty} y(t) = -2$.

(iii) $\lim_{t\to\infty} y(t) = 0$.

(iv) $\lim_{t\to\infty} y(t) = 2$.

(v) $\lim_{t\to\infty} y(t) = \infty$.

(c) If $y(t)$ is the solution of the initial value problem $y' = y^3 - 4y,\ y(1) = 2$, then

(i) $\lim_{t\to\infty} y(t) = -\infty$.

(ii) $\lim_{t\to\infty} y(t) = -2$.

(iii) $\lim_{t\to\infty} y(t) = 0$.

(iv) $\lim_{t\to\infty} y(t) = 2$.

(v) $\lim_{t\to\infty} y(t) = \infty$.

YOUR SIGNATURE:

(3) Consider the initial value problem

$$\cos(t)\frac{dy}{dt} - y\sin(t) = \frac{1}{t^2}, \quad y(\pi) = -\frac{2}{\pi}.$$

(a) Find an explicit formula for $y(t)$.

(b) Find the interval of existence for this solution.

YOUR SIGNATURE:

(4) Consider the differential equation

$$\left(1 + ye^{xy}\right)dx + \left(2y + xe^{xy}\right)dy = 0.$$

(a) Show that the differential equation is exact.

(b) Find the implicit solution of the differential equation.

(5) (a) Use the substitution $y = t^r$ to find two solutions y_1 and y_2 of the homogeneous equation

$$t^2 y'' + t y' - y = 0, \quad t > 0.$$

(b) Use the Wronksian to verify that y_1 and y_2 are linearly independent.

(c) (Continued from the previous page) Use the **variation of parameters** method to find the general solution of

$$t^2y'' + ty' - y = \frac{1}{t^2}, \quad t > 0$$

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(6) Use the method of undetermined coefficients to find a general solution of

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x + 1 + e^{3x}.$$

(7) A bacterium population $P(t)$, grows according to the logistic differential equation

$$\frac{dP}{dt} = r_0\big(1 - P/2000\big)P.$$

Suppose the initial population is 25% of the carrying capacity, and the population doubles after 2 hours. Find the number of bacteria present after 20 hours.

YOUR SIGNATURE:

FORMULA SHEET

(1) **Integration By Parts:** $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$.

(2) **Partial Fractions Integral:** If $c \neq d$ then

$$\int \frac{ax+b}{(x-c)(x-d)}dx = \frac{1}{c-d}\big[(ac+b)\ln|x-c| - (ad+b)\ln|x-d|\big] + K.$$

(3) **The Logistic Equation:** $P' = r_0(1 - P/K)P$ has the implicit general solution

$$\frac{P}{K-P} = \frac{P_0}{K-P_0}e^{r_0 t}.$$

(4) **For linear homogeneous d.e. with constant coefficients:** $y'' + by' + cy = 0$.

- If $b^2 - 4c > 0$, then r_1 and r_2 are two distinct solutions of the characteristic equation and
$$y = C_1e^{r_1t} + C_2e^{r_2t},$$
where C_1 and C_2 are constants.
- If $b^2 - 4c = 0$, then there is only one solution of the characteristic equation, $r = -b/2$, and
$$y = C_1te^{rt} + C_2e^{rt}.$$
- If $b^2 - 4c < 0$, then the solutions of the characteristic equation are of the form $r = \alpha \pm \beta i$ and
$$y = C_1e^{\alpha t}\cos(\beta t) + C_2e^{\alpha t}\sin(\beta t).$$

(5) **For linear non-homogeneous d.e. with constant coefficients:**

If $f(x)$ is	then try $y_p(x)$ in the form of
polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0$	polynomial of same degree $A_nx^n + A_{n-1}x^{n-1} + \cdots + A_0$
be^{kx}	Be^{kx}
$b\sin(ax)$ or $b\cos(ax)$	$B\sin(ax) + C\cos(ax)$

(6) **Variation of Parameters:** If y_1 and y_2 are linearly independent solutions of the equation $y'' + p(t)y' + q(t)y = 0$, then $y_p = v_1y_1 + v_2y_2$ is a particular solution of the equation $y'' + p(t)y' + q(t)y = f(t)$, where v_1 and v_2 satisfy the VOP equations

$$\begin{aligned} v_1'y_1 + v_2'y_2 &= 0 \\ v_1'y_1' + v_2'y_2' &= f(t). \end{aligned}$$