Polytechnic University

MA 2132 MIDTERM JANUARY 9TH, 2007

Print Name:

Signature:

ID #:

Instructor/Section: Andy Tsang

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator, **but you must show your work for integrals and derivatives**.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID, and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

YOUR SIGNATURE:

(1) (Page 41, Problem 25) Find the solution of the initial value problem:

$$x\frac{dy}{dx} - y = 2x^2y, \quad y(1) = 1$$

Show all your work.

YOUR SIGNATURE:

(2) (Page 71, Problem 22) Find the general solution of the differential equation:

$$x^2y' + 2xy = 5y^4$$

Show all your work.

(3) (Worksheet 2, Problem 1) Suppose y satisfies the following autonomous differential equation:

$$\frac{dy}{dt} = y^3 - 6y^2 + 8y$$

Answer the following questions. You do NOT need to explain your answer.

(a) If $y(3) = 1$, then $\lim_{t\to\infty} y(t) =$ _____

(b) If $y(1) = 3$, then $\lim_{t\to\infty} y(t) =$ _____

(c) If $y(0) = 5$, then $\lim_{t\to-\infty} y(t) =$ _____

(d) If $y(0) = 4$, then $y(2) =$ _____

(e) If $y(1) = 0$, then $y(2) =$ _____

YOUR SIGNATURE:

(4) (Worksheet 3, Problem 2) Find the solution of the initial value problem:

$$y'' - 2y' + y = 3e^x, \quad y(0) = 1, \quad y'(0) = 1$$

Show all of your work.

YOUR SIGNATURE:

(5) (Page 347, Problem 53) Find the general solution of the differential equation:

$$y'' + 9y = \frac{2}{\cos(3x)}$$

Show all of your work.

YOUR SIGNATURE:

FORMULA SHEET

(a) **Integration By Parts:** $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx.$

(b) **Partial Fractions Integral:** If $c \neq d$ then

$$\int \frac{ax+b}{(x-c)(x-d)}dx = \frac{1}{c-d}\big[(ac+b)\ln|x-c| - (ad+b)\ln|x-d|\big] + K.$$

(c) **The Logistic Equation:** $P' = r_0(1 - \frac{P}{K})P$ has the implicit general solution

$$\frac{P}{K-P} = \frac{P_0}{K-P_0}e^{r_0 t}$$

(d) **Variation of Parameters:** If y_1 and y_2 are linearly independent solutions of the equation $y'' + p(t)y' + q(t)y = 0$, then $y_p = v_1y_1 + v_2y_2$ is a particular solution of the equation $y'' + p(t)y' + q(t)y = f(t)$, where v_1 and v_2 satisfy the VOP equations

$$\begin{aligned} v_1'y_1 + v_2'y_2 &= 0 \\ v_1'y_1' + v_2'y_2' &= f(t). \end{aligned}$$