Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator, but you must show your work for integrals and derivatives. If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID, and accept any written statement(s) that you may wish to make regarding your illness.

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<th>Problem</th>
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</table>
(1) (Page 41, Problem 25) Find the solution of the initial value problem:

\[ x \frac{dy}{dx} - y = 2x^2 y, \quad y(1) = 1 \]

Show all your work.
(2) (Page 71, Problem 22) Find the general solution of the differential equation:

\[ x^2y' + 2xy = 5y^4 \]

Show all your work.
(3) (Worksheet 2, Problem 1) Suppose \( y \) satisfies the following autonomous differential equation:

\[
\frac{dy}{dt} = y^3 - 6y^2 + 8y
\]

Answer the following questions. You do NOT need to explain your answer.

(a) If \( y(3) = 1 \), then \( \lim_{t \to \infty} y(t) = \) 

(b) If \( y(1) = 3 \), then \( \lim_{t \to \infty} y(t) = \) 

(c) If \( y(0) = 5 \), then \( \lim_{t \to -\infty} y(t) = \) 

(d) If \( y(0) = 4 \), then \( y(2) = \) 

(e) If \( y(1) = 0 \), then \( y(2) = \)
(4) (Worksheet 3, Problem 2) Find the solution of the initial value problem:

\[ y'' - 2y' + y = 3e^x, \quad y(0) = 1, \quad y'(0) = 1 \]

Show all of your work.
(5) (Page 347, Problem 53) Find the general solution of the differential equation:

\[ y'' + 9y = \frac{2}{\cos(3x)} \]

Show all of your work.
FORMULA SHEET

(a) **Integration By Parts:** \[ \int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx. \]

(b) **Partial Fractions Integral:** If \( c \neq d \) then
\[
\int \frac{ax + b}{(x - c)(x - d)} \, dx = \frac{1}{c - d} \left[ (ac + b) \ln |x - c| - (ad + b) \ln |x - d| \right] + K.
\]

(c) **The Logistic Equation:** \( P' = r_0(1 - \frac{P}{K})P \) has the implicit general solution
\[
\frac{P}{K - P} = \frac{P_0}{K - P_0} e^{r_0 t}.
\]

(d) **Variation of Parameters:** If \( y_1 \) and \( y_2 \) are linearly independent solutions of the equation \( y'' + p(t)y' + q(t)y = 0 \), then \( y_p = v_1 y_1 + v_2 y_2 \) is a particular solution of the equation \( y'' + p(t)y' + q(t)y = f(t) \), where \( v_1 \) and \( v_2 \) satisfy the VOP equations
\[
\begin{align*}
v_1' y_1 + v_2' y_2 &= 0 \\
v_1' y_1' + v_2' y_2' &= f(t).
\end{align*}
\]