Polytechnic University

MA 2222 FINAL DECEMBER 14, 2004

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	18	
2	18	
3	16	
4	16	
5	16	
6	16	
Total	100	

(1) (18 points)

(a) In testing the hypothesis $H_0 : \theta = \theta_0$ against the simple alternative hypothesis $H_1 : \theta > \theta_0$ what happens to β, the probability of committing a type II error, when α, the level of significance, is reduced? (Assume that this is the only change.)

(i) β is reduced in size.
(ii) β is increased in size.
(iii) β is unchanged.
(iv) The answer depends on the initial value of α.
(v) The answer depends on the form of the null hypothesis.

(b) Two samples one of size 1000 the other of size 1200, were taken to compare two proportions. If the null hypothesis H_0 is $p_1 = p_2$, and the alternative hypothesis H_1 is $p_1 > p_2$, and the computed statistic equals 1.96, which of the following is the correct p-value?

(i) 0.050.
(ii) 0.0250.
(iii) 0.0495.
(iv) another value.

(c) The confidence interval, obtained with a small sample $\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}(n-1)}\right]$ for a population variance is obtained from a population having

(i) any distribution with finite mean and variance.
(ii) a t-distribution.
(iii) a normal distribution.
(iv) a binomial distribution.
(v) none of these distributions.

(2) (18 points) Corn was grown in two different kinds of soils A and B and the heights of the stalks were measured. A sample of 10 stalks from soil A gave a mean height of 6.4 ft and a standard deviation of 1.2 ft, whereas a sample of 16 stalks from soil B gave a mean height of 6.3 ft and a standard deviation of 1.1 ft. Assume the heights to be normally distributed.

(a) Test for equality of variances at the 0.05 level of significance.

(b) Give a 98% confidence interval for the difference in the means. Are the means significantly different at this level? Why or why not?

(3) (16 point) Different methods of instruction were administered to three groups of students I, II and III and their performances, as rated by their grades A to F, were tabulated below:

Grade	A	B	C	D	F	Totals
Method I	4	5	4	4	3	20
Method II	6	7	9	7	6	35
Method III	8	7	7	7	6	35
Totals	18	19	20	18	15	90

Is there any evidence at the 0.05 level of significance that there is any difference in efficiency in the three methods?

(4) (16 points) Let μ_i be the average yield in bushels per acre of varieties i of wheat, $i = 1, 2, 3$. In order to test at the 5% significance level, the hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$, four test plots for each of the four varieties of wheat were planted. Assume normal distributions for the X_i. Determine whether we accept or reject H_0 if the yield in bushels per acre of the four varieties of wheat were:

X_1 :	68	76	74
X_2 :	86	75	77
X_3 :	90	78	80

Recall that

$$\frac{SS(T)/(m-1)}{SS(E)/(n-m)}$$

has an F distribution with $m - 1$ and $n - m$ degrees of freedom; and also that

$$SS(T) = \sum_{i=1}^{m} n_i(\overline{X_{i.}} - \overline{X_{..}})^2,$$

$$SS(E) = \sum_{i=1}^{m}\sum_{j=1}^{n_i}(X_{ij} - \overline{X_{i.}})^2.$$

(5) (16 points) A random sample of 9 tunafish yielded the following observations of their lengths (in ft):

5.1 4.0 5.3 5.6 2.9 6.2 6.5 2.7 3.7

Using the Wilcoxon test, test at the 1% confidence level the hypothesis that the median $m = 3.8$ against the alternative hypothesis that $m > 3.8$. Recall that W, the Wilcoxon statistic is approximately normally distributed with mean 0 and variance $\dfrac{n(n+1)(2n+1)}{6}$.

(6) (16 points) Let$X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from the normal distribution $N(\mu, \sigma^2)$.

(a) Find the critical region C of size $\alpha = 0.05$ for testing $H_0 : \sigma^2 = 30$ against $H_1 : \sigma^2 = 120$.

(b) Find, approximately, the appropriate value of β, the probability of a type II error, for the critical region C of part (a). (Assuming $H_1 : \sigma^2 = 120$.)