Polytechnic University

MA 2222 MIDTERM APRIL 15, 2002

Print Name:

Signature:

ID #:

Directions: You have two hours to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

Problem	Possible	Points
1	20	
2	20	
3	20	
4	15	
5	15	
6	10	
Total	100	

(1) (20 points, Problem 4 on Worksheet 4) A company uses 2 machines to fill bags of dog food. For a sample of 13 bags filled by the newer machine, the mean was 81.7 ounces and the standard deviation was 1.3 ounces. For a sample of 16 bags filled by the older machine, it had a mean of 81.0 ounces and a standard deviation of 2.1 ounces. Assume both populations have approximately normal distributions.

(a) Is there sufficient evidence at $\alpha = 0.05$ level to conclude that the variance of the amounts dispensed by the old machine is greater than that of the newer machine? State the hypothesis H_o and H_a, carry out the appropriate test, and state your conclusion clearly.

(b) Test if the means of the amounts dispensed by the two machines are equal at significance level $\alpha = 0.05$.

(2) (20 points, Web-homework for Section 8.1, Problem 2) The proportion of adults living in a small town who are college graduates is claimed to be $p = 0.6$. To test this claim, a random sample of 15 adults is selected. If the number of college graduates in our sample is anywhere from 6 to 12, we will accept the null hypothesis that $p = 0.6$; otherwise, we shall conclude that $p \neq 0.6$.

(a) Use the binomial distribution to evaluate the type I error α.

(b) Evaluate type II error β for the alternative hypothesis $p = 0.7$.

(c) Repeat part (b) when 200 adults are selected and the *acceptance region* is defined to be $110 \leq x \leq 130$ where x is the number of college graduates in the sample. Please use normal distribution to approximate the appropriate probability for computing β.

(3) (20 points, Problems 1,2 on Worksheet 3 and Web-homework for Section 7.6) An economist wants to estimate the average hourly wage of waitresses in a large metropolitan area. He will do this by randomly surveying 225 waitresses and calculating the sample mean. Then he will use the sample mean as his estimate.

(a) Find the probability that the estimate will be in error by more than 5 cents. Assume that the standard deviation of the hourly wage of waitresses in this metropolitan area is 45 cents.

(b) What is the sample size that the economist needs to survey so that he can be 95% confident that his estimate will be off from the actual mean by at most 5 cents? Again, assume that the standard deviation is 45 cents.

(c) Do we need the assumption of normality on the distribution of the hourly wage of waitresses in this metropolitan area? Explain!

(4) (15 points, Problem 3 on Worksheet 4.) There was a 1987 Gallup survey (reported April 12, 1987) on whether "there is evidence of growing public intolerance towards gays." The poll used a sample of 1015 adults from "scientifically selected localities across the nation." One question asked was this: "Do you think homosexual relations between consenting adults should or should not be legal?" In 1986 the percentage who said "should not" was 54; in the 1987 poll, it was 55. Assume that both samples from 1986 and 1987 are random, and that the 1986 poll was based on the same number of people. Is this enough evidence of growing intolerance?

(**Hint:** You can use whatever method you choose to answer the question, but make sure you state clearly all the hypotheses, computation, conclusion and reasoning.)

(5) (15 points, MA 2212 material and Problem 3 on Worksheet 1) Let X be an exponential distribution with probability density function (p.d.f.)

$$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Given a positive constant θ, find $P(X < x | X < \theta)$ for $0 < x < \theta$. (This is the cumulative distribution function (c.d.f.) of X if we truncate the distribution at θ.)

(b) Using the following 10 random observations from a distribution with p.d.f.

$$f(x) = \frac{e^{-x}}{1 - e^{-\theta}}, \qquad 0 < x < \theta,$$

find the maximum likelihood estimate of θ.

1.46 0.22 0.80 0.42 1.45 0.84 1.89 1.27 0.49 0.55

(6) (10 points, Problem 3 on Worksheet 2) It can be shown that for a random sample of size n from a exponential distribution with mean θ, the quantity

$$\frac{2}{\theta}\sum_{i=1}^{n} X_i$$

is distributed as Chi-square distribution with $r = 2n$ degrees of freedom. (You don't need to prove it now, just accept it as a known result.) Use this fact to obtain a 90% confidence interval for θ if $n = 10$ and $\sum_{i=1}^{10} X_i = 32$.

A table for some well-known distributions. (Note: In the following table $q = 1 - p$.)

Name	p.d.f	m.g.f	mean	variance
Bernoulli(p)	$f(x) = p^x q^{1-x}, \quad x = 1, 2$	$M(t) = q + pe^t$	p	pq
Binomial(n, p)	$f(x) = \binom{n}{x} p^x q^{n-x}$ $x = 0, 1, 2, \dots, n$	$M(t) = (q + pe^t)^n$	np	npq
Geometric(p)	$f(x) = q^{x-1} p$ $x = 1, 2, \dots$	$M(t) = \frac{pe^t}{1 - qe^t}$	$\frac{1}{p}$	$\frac{q}{p^2}$
Negative Binomial (r, p)	$f(x) = \binom{x-1}{r-1} p^r q^{x-r}$ $x = r, r+1, r+2, \dots$	$M(t) = \frac{(pe^t)^r}{(1 - qe^t)^r}$	$\frac{r}{p}$	$\frac{rq}{p^2}$
Poisson(λ)	$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$ $x = 0, 1, 2, \dots$	$M(t) = e^{\lambda(e^t - 1)}$	λ	λ
Exponential(θ)	$f(x) = \frac{1}{\theta} e^{-x/\theta}$ $0 \leq x < \infty$	$M(t) = \frac{1}{1 - \theta t}$	θ	θ^2
Gamma (α, θ)	$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}$ $0 \leq x < \infty$	$M(t) = \frac{1}{(1 - \theta t)^\alpha}$	$\alpha\theta$	$\alpha\theta^2$
Chi-Square(r)	$f(x) = \frac{1}{\Gamma(\frac{r}{2}) 2^{\frac{r}{2}}} x^{r/2-1} e^{-x/2}$ $0 \leq x < \infty$	$M(t) = \frac{1}{(1 - 2t)^{r/2}}$	r	$2r$
Normal $N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$ $-\infty < x < \infty$	$M(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$	μ	σ^2