Polytechnic University

MA 2222 MIDTERM NOVEMBER 24, 2004

Print Name:

Signature:

ID #:

Instructor:

Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	18	
2	16	
3	16	
4	18	
5	16	
6	16	
Total	100	

(1) (18 points) Fill in the blanks.

(a) Let X_1,X_2,...,X_9 be a random sample from the standard normal distribution N(0,1). If

$$W = X_1^2 + X_2^2 + \cdots + X_9^2,$$

then

$$P(2.7 < W < 14.68) = \_\_\_\_\_\_\_\_\_\_.$$

(b) Let Z be N(0,1) and W as in part (a). If $U = \dfrac{Z}{\sqrt{W}}$, then

$$P(-0.234 < U < 0.940) = \_\_\_\_\_\_\_\_\_\_.$$

(c) Let $Y_1, Y_2, \ldots Y_{12}$ be a random sample from another N(0,1) population. If

$$V = Y_1^2 + Y_2^2 + \cdots + Y_{12}^2$$

and $G = \dfrac{V}{W}$ (W as in part (a)), then

$$P\left(\frac{1}{2.58} < G < 4.09\right) = \_\_\_\_\_\_\_\_\_\_.$$

(2) (16 points) Let X_1,X_2,...,X_n denote a random sample of size n from the distribution with p.d.f

$$f(x;\theta) = \frac{1}{2\theta^3} x^2 e^{-\frac{x}{\theta}}$$

for $x \geq 0$ and 0 elsewhere, where the parameter θ is > 0.

(a) Find the maximum likelihood estimator for θ.

(b) Find the method of moments estimator for θ. [Hint: $\int_0^\infty y^n e^{-y} dy = n!$]

(c) For the following six observations, calculate the values of the maximum likelihood estimate and the method of moments estimate for θ:
1.47 1.53 1.46 1.54 1.45 1.55.

(3) (16 points)

(a) Give a 95% confidence interval for the proportion of voters voting for a candidate given that a sample of 1000 voters gave 525 favorable votes. Show your work.

(b) Is the candidate 95% sure of being elected? Justify your answer.

(c) How large should the sample be in order that the error of the estimate be less than 2%? Show your work.

(4) (18 points) Corn plants were grown on two patches: patch A with natural soil, patch B with a certain fertilizer. The lengths of the stalks were measured after 3 months. Assume normal distributions for the stalk sizes. For a sample of size 12 from patch A, the observed mean was found to be 3.5 ft and the observed sample standard deviation to be 1.2 ft. For a sample of size 15 from patch B the observed mean was found to be 4.0 ft and the sample standard deviation to be 1.5 ft.

(a) Test the hypothesis that the variances are equal at the 5% significance level.

(b) On the basis of your result in (a) find a 95% confidence interval for the difference of the means. Can you be 95% confident that the means are equal. Explain.

(5) (16 points) Two computers are compared by feeding them a collection of benchmark programs and recording the differences in times required to complete the execution of these programs. Six benchmark programs, run on the two computers produced the following running times. Assume the running times for both computers to be normally distributed.

Program	1	2	3	4	5	6
Computer 1	2.12	2.73	2.04	2.86	2.47	3.10
Computer 2	2.15	2.72	2.10	2.87	2.46	3.15

(a) Are the running times of the benchmark programs for the two computers independent? Explain.

(b) Does the data provide sufficient evidence at the significance level $\alpha = 0.05$ to indicate a difference in mean running times for the two computers?

(6) (16 points) There might be evidence that some fertilizer gives bigger potatoes. The average weight of untreated potatoes is 3 oz. The manufacturer of the fertilizer claims that this average weight is 3.5 oz with the fertilizer. We wish to set up a test of hypothesis at the 0.05 level, with a sample of size 16. Assume normal distribution for the weights.

(a) What would be H_0 and H_1?

(b) What would be the test statistic? What would be its distribution?

(c) Suppose the sample mean was found to be 3.3 oz and the sample standard deviation was found to be 0.7 oz, what is your conclusion? Give the p-value.