Polytechnic University

MA 2222 MIDTERM NOVEMBER 16, 2005

Print Name:

Signature:

ID #:

instructor:

Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

Problem	Possible	Points
1	15	
2	15	
3	15	
4	15	
5	20	
6	10	
7	10	
Total	100	

(1) (15 points) Let X be a random variable with p.d.f. given by

$$\begin{aligned} f(x,\theta) &= (1+\theta)x^{\theta} \ for \ 0 \leq x \leq 1 \\ &= 0 \ elsewhere \end{aligned}$$

where θ is a parameter with $-1 < \theta < \infty$.

(a) Show $E(X) = (1+\theta)/(2+\theta)$

(b) What is the expectation of the sample mean $\overline{X}$?

(c) Find the estimator determined by the methods of moments (i.e., solve for θ in terms of $\overline{X}$).

(d) What is the likelihood function, $L(\theta)$? What is the log likelihood function ?

(e) Find the maximum likelihood estimator, called $\hat{\theta}$, for θ.

(2) (15 points) Give the explicit the middle 80% probability interval for the sample variance, s^2, for a random sampling of n values, say $X_1, X_2, \cdots, X_n$, from a normal distribution with mean $\mu = 100$ and variance $\sigma^2 = 5$.

(a) For $n = 10$: __________ to __________ (three decimal places)

(b) For $n = 20$: __________ to __________ (three decimal places)

(c) For $n = 30$: __________ to __________ (three decimal places)

(3) (15 points)

(a) Give the explicit the 56% confidence interval for the mean, given a random sampling of n values, say $X_1, X_2, \cdots, X_n$, from a normal distribution with sample mean $\overline{X} = 100$ and variance $\sigma^2 = 5$. Suppose that $n = 10$.

Answer: ____________ to ____________ (three decimal places)

(b) Let instead $n = 20$, but we desire a 90% confidence interval for the mean, given a random sampling of n values, say $X_1, X_2, \cdots, X_n$, from a normal distribution with sample mean $\overline{X} = 100$ and sample variance $s^2 = 5$.

Answer: ____________ to ____________ (three decimal places)

(c) Let instead $n = 120$, but we desire a 95% confidence interval for the mean, given a random sampling of n values, say $X_1, X_2, \cdots, X_n$, from a normal distribution with sample mean $\overline{X} = 100$ and sample variance $s^2 = 5$.

Answer: ____________ to ____________ (three decimal places)

(4) (15 points) The p.d.f. for a distribution is

$$f(x,\beta) \quad = (\tfrac{\beta}{\beta-1}) \;\; x^{-2} \; for \; 1 < x < \beta$$
$$= 0 \; elsewhere$$

where the parameter β is $1 < \beta < \infty$.

Let $X_1, X_2, \cdots, X_n$ be a random sample from this distribution.

(a) What is a maximum likelihood function $L(\beta)$?

(b) Show that $L(\beta)$ [or equivalently $ln\ L(\beta)$] is decreasing.

(c) Show that the Maximum Likelihood Estimator for β is the n^{th} order statistic, $Y_n = max(X_1, X_2, \cdots, X_n)$.

(d) Suppose that $n = 2$. Find the expectation $E[Y_2]$ in terms of β.

(5) (20 points) In a public opinion poll for a close presidential election, let p denote the proportion of voters who favor candidate A. If in a sample of 2000, there were 1050 who favored the candidate A.

(a) Discuss the chances candidate A will win the election on the basis of the sample evidence. Provide a #.

(b) How large a sample should be taken if we want the maximum error of the estimate of p, (i.e., half the length of the confidence interval), to be equal to 0.02 with 95% confidence?

Answer: $n \geq$ ____________.(an integer)

(c) W hat is a 95% confidence interval for the proportion of people who favor candidate A.

Answer: ____________ to ____________ (to 3 decimal places)

(6) (10 points) Let X, Y given be the number of mice discovered at two different factories in Brooklyn during the first 10 months of 2005:

	Jan	*Feb*	*Mar*	*April*	*May*	*June*	*July*	*Aug*	*Sept*	*Oct*
X	5	7	9	12	15	20	40	35	30	25
Y	6	8	8	13	16	18	36	32	28	26

with sample means $\bar{X} = 19.80, \bar{Y} = 19.10$ and sample standard deviations $s_X = 12.263, s_Y = 10.774$ respectively.

(a) Find a 95% confidence interval for the difference in means.

Answer: ____________ to ____________(three decimals)

(b) Is there a significant differnce in the number of mice in these two factories? Why or why not ?

(7) (10 points) For a sample of 13 from a batch of cooling chocolate the average sugar content was 3.26 grams with a sample standard deviation of 1.2 grams. For a second sample of 16 from another batch the average sugar content was found to be 4.31 grams with a sample standard deviation of 2.3 grams.

(a) Find a 90% confidence interval for the ratio of variances.

Answer:____________to ____________

(b) Is there a significant reason to assume the two variances are equal?

(c) Do you find a significant difference in the means of these two samples ? [Justify the formula used, Welch's or whatever!!!]