# Polytechnic University

MA 2312 FINAL March 10th, 2003

Print Name:

Signature:

ID #:

Instructor/Section: Cornick

**Directions:** You have **90 minutes** to answer the following questions. Unless otherwise indicated, you MUST show all your work to receive credit. Please put a tick in the upper right hand corner of this page to indicate that you have read the directions. No calculators.

| Problem | Possible | Points |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 16 | |
| 4 | 16 | |
| 5 | 16 | |
| 6 | 16 | |
| 7 | 16 | |
| Total | 100 | |

(1) (10 points) Let $A$ and $B$ be FINITE (non-empty) sets, let $f : A \to B$ be a function and let $f(A)$ denote the image of $f$. State whether the following are **TRUE** or **FALSE**. (You do not need to explain your answer.)

(a) If $|A| \geq |B|$ then $f$ is surjective.

(b) If $f$ is surjective then $|A| \geq |B|$.

(c) If $|A| \leq |B|$ then $f$ is injective.

(d) If $f$ is injective then $|A| \leq |B|$.

(e) If $|A| = |f(A)|$ then $f$ is injective.

(f) If $f$ is injective then $|A| = |f(A)|$.

(g) $f$ is bijective only if $|A| = |B|$.

(h) If $|f(A)| = |B|$ then $f$ is surjective.

(i) If $|f(A)| > |B|$ then $f$ is injective.

(j) $f$ is injective if and only if $f$ is surjective.

(2) (10 points) State whether the following are Tautologies (logically equivalent to **T**), Contradictions (logically equivalent to **F**), or Neither. (You do not need to explain your answer.)

(a) $(\mathbf{F} \to \mathbf{T}) \to \mathbf{F}$

(b) $\mathbf{F} \to (\mathbf{T} \to \mathbf{F})$

(c) $(\mathbf{F} \to p) \to \neg p$

(d) $(p \to \mathbf{F}) \to \neg p$

(e) $((p \to q) \wedge (\neg p \to q)) \to q$

(f) $((p \to q) \wedge (p \to \neg q)) \to p$

(g) $(p \to q) \vee (q \to p)$

(h) $\neg(p \to q) \wedge (\neg q \to \neg p)$

(i) $(p \to \neg p) \wedge p$

(j) $p \to (\neg p \wedge p)$

(3) (Worksheet 5) Let $m = d_n d_{n-1} \dots d_3 d_2 d_1 d_0$ be an $n$-digit integer. (So for example 1287 has $d_0 = 7, d_1 = 8, d_2 = 2, d_4 = 1$.)

(a) Prove that $m \equiv (\sum_{i=0}^{n} d_i) \mod 9$.

(b) Use the above result to determine the remainder $r$ (where $0 \leq r < 9$), when the following numbers are divided by 9. (No credit for using long division, negative credit for using a calculator.)

(i) 121314151617

(ii) 12345678987654321

(4) (Worksheet 4) Let $a, b$ be positive integers, and let $p$ be a prime number.

(a) Show that if there exists integers $s$ and $t$ such that $as+bt = p$ then $\gcd(a, b) = 1$ or $\gcd(a, b) = p$. (Hint: Find the possible common divisors of $a$ and $b$.)

(b) Find integers $s$ and $t$ such that $40s + 99t = 1$.

(c) Find integers $u$ and $v$ such that $40u + 99v = 17$.

(5) (Worksheet 3) Let $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$ denote the set of natural numbers.

(a) Define what it means for two sets $A$ and $B$ to have the same cardinality.

(b) Define what it means for a set $A$ to be countable.

(c) Prove that the set of integers which are perfect squares is a countable set using the above definition.

(d) Prove that the set of odd integers is a countable set using the above definition.

(6) (Worksheet 5) Prove the following using mathematical induction.

(a) If $n$ is a non-negative integer then $n^2 + n$ is an even integer.

(b) If $n$ is a non-negative integer then $6|(n^3 - n)$.

(7) (Worksheet 2) Consider the two propositions $\exists x(P(x) \wedge Q(x))$ and $\exists x P(x) \wedge \exists x Q(x)$ where $P(x)$ and $Q(x)$ are propositional functions, and $x$ belongs to some Universe of Discourse.

(a) Decide which of these propositions always implies the other, and then prove your claim carefully.

(b) Find an example where one of the propositions is true and the other is false. Explain your answer carefully.

(c) (5 points - extra credit.) True or False: (No points for guessing, you will only get credit for a clear and correct explanation.)

$$\forall A \exists P(x)(\forall x \in A(P(x)) \Rightarrow \exists x \in A(P(x)))$$

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