Polytechnic University

MA 2312 FINAL JUNE 23, 2003

Print Name:
Signature:
ID #:
Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. Unless otherwise indicated, **you MUST show all your work to receive credit.** Please put a tick in the upper right hand corner of this page to indicate that you have read the directions. No calculators.

Problem	Possible	Points
1	10	
2	10	
3	16	
4	16	
5	16	
6	16	
7	16	
Total	100	

(1) (10 points) A function $f : \mathbf{R} \to \mathbf{R}$ is said to be *strictly decreasing* if

$$\forall x \forall y (y > x \to f(y) < f(x)).$$

Which of the following are logically equivalent to the definition of a strictly decreasing function. You do not need to explain.

(a) $\forall x (f'(x) < 0)$

(b) $\forall x \forall y (x > y \to f(x) < f(y))$

(c) $\forall x \forall y (y \leq x \to f(y) \geq f(x))$

(d) $\forall x \forall y (f(y) \geq f(x) \to y \leq x)$

(e) $\forall x \exists y (x > y \wedge f(x) < f(y))$

(2) (10 points) Let B be a Boolean Algebra and let x, y and z be elements of B. State whether the following are Identities (indicate with **I**), true for Some Values of x, y and z (indicate with **SV**) or Never True (indicate with **NT**). You do not need to explain.

(a) $x \cdot (x + y) = x$.

(b) $x \cdot (x + y) = \overline{y}$.

(c) $(x + y) \cdot z = x + y \cdot z$.

(d) $(x + \overline{y}) + z = y + (\overline{x} + \overline{z})$.

(e) $x + x \cdot y = x$.

(3) (16 points) Let $m = d_n d_{n-1} \ldots d_3 d_2 d_1 d_0$ be an $n+1$-digit integer. (So for example 1287 has $d_0 = 7, d_1 = 8, d_2 = 2, d_3 = 1$.)

(a) Prove carefully that $m \equiv \left(\sum_{i=0}^{n} (-1)^i d_i\right) \mod 11$.

(b) Use the above result to determine the remainder r (where $0 \leq r < 11$), when the following numbers are divided by 11. (No credit for using long division or just writing down the answer, negative credit for using a calculator.) You must show your work.

(i) 98897667544532231001

(ii) 1101101101101101

(4) (16 points)

(a) Find two integers s and t such that $25s + 36t = 1$.

(b) Solve $25x \equiv 3 \mod 36$.

(5) (16 points) Let $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$ denote the set of natural numbers.

(a) Define what it means for two sets A and B to have the same cardinality. (Your definition should include a function.)

(b) Define what it means for a set A to be countable.

(c) Prove that the set of natural numbers which are divisible by 4 is a countable set.

(d) Prove that the set of natural numbers which are not divisible by 4 is a countable set.

(6) (16 points) Prove by induction that a set with n elements has 2^n subsets.

(7) (16 points) Let $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$ be the Universe of Discourse in this question.

(a) Find an example of a propositional function $P(n)$ such that the proposition $P(0)$ is true but the proposition $\forall n P(n)$ is false. Explain.

(b) Find an example of a propositional function $P(n)$ such that the proposition $\forall n(P(n) \rightarrow P(n+1))$ is true but the proposition $\forall n P(n)$ is false. Explain.