

Polytechnic University

MA 2312

MIDTERM

FEBRUARY 18TH, 2003

Print Name:
Signature:
ID #:
Instructor/Section: Cornick

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. No calculators.

Problem	Possible	Points
1	10	
2	10	
3	16	
4	16	
5	16	
6	16	
7	16	
Total	100	

(1) (10 points) State whether the following are **TRUE** or **FALSE**. (You do not need to explain your answer)

(a) $|P(\{\emptyset\})| = 2$

(b) $\emptyset \cup \{\emptyset\} = \{\emptyset\}$

(c) $\{1, 2\} \in \{1, 2, 3\}$

(d) $\{1, 2\} \subseteq P(\{1, 2, 3\})$

(e) $\{1, 2\} \in P(\{1, 2, 3\})$

(f) $(\mathbf{R} \times \mathbf{Z}) \cap \mathbf{R} = \mathbf{R}$

(g) $p \rightarrow (p \wedge q)$ is a tautology.

(h) A proposition is logically equivalent to its converse.

(i) A proposition is logically equivalent to its contrapositive.

(j) "If this exam has an extra credit problem then everyone will pass."

(2) (10 points) State whether the following are propositions, propositional functions, or neither. (You do not need to explain your answer)

(a) $x \in \mathbf{Z}$

(b) $\pi \in \mathbf{Z}$

(c) $3 \Rightarrow \mathbf{Z}$

(d) $\mathbf{R} \subseteq \mathbf{Z}$

(e) $\mathbf{Z} \cap \mathbf{R}$

(f) $\mathbf{Z} \cup A = \mathbf{R}$

(g) $\forall x \in \mathbf{R}(x \neq x)$

(h) "If the 2312 midterm is on Wednesday, then $1+1=3$ "

(i) "When will the exam be graded?"

(j) $x + y$

(3) (15 points - Worksheet 3) Consider the intervals $A = (0, 1)$ and $B = (-\infty, 3)$.

(a) Find a bijective function $f : A \rightarrow B$. Explain carefully why it is bijective.

(b) Find a formula for the inverse function $f^{-1} : B \rightarrow A$.

(4) (15 points - Worksheet 2) Let A , B and C be subsets of a universal set U .

(a) Draw Venn diagrams for the sets $A \cap (B - C)$ and $A - (B \cap C)$.

(b) Decide which of the sets $A \cap (B - C)$ and $A - (B \cap C)$ is a subset of the other, and then prove your claim carefully.

(5) (10 points - Worksheet 3) Plot the graphs of the following functions.

(a) $f : \mathbf{N} \rightarrow \mathbf{R}$ defined by $f(n) = -\sqrt{n}$ for $n \in \mathbf{N}$.

(b) $g : \mathbf{R} \rightarrow \mathbf{Z}$ defined by $g(x) = \lfloor x \rfloor - \lceil x \rceil$ for $x \in \mathbf{R}$.

(6) (15 points - Worksheet 2) Consider the two propositions $\exists x\forall yP(x, y)$ and $\forall y\exists xP(x, y)$ where $P(x, y)$ is any propositional function.

(a) Decide which of these propositions always implies the other, and then prove your claim carefully.

(b) Find an example where one of the propositions is true and the other is false. Explain your answer.

(7) (15 points - Worksheet 1) There are two kinds of natives on a remote island - Knights who always tell the truth, and Knaves who always lie. It is impossible to distinguish between knights and knaves visually. You meet three natives A, B and C on the island: First A says "I am a knight", then B says "A is lying" and then C says "B is lying".

(a) What conclusions can you draw about the identities of A, B and C?

(b) After A, B and C have made their statements, A then says "I am the opposite kind of native to C". What are A, B and C?