Polytechnic University

MA 2312 MIDTERM JUNE 12, 2003

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You will **NOT** receive full credit for a correct answer without explanation. No calculators.

Problem	Possible	Points
1	10	
2	10	
3	16	
4	16	
5	16	
6	16	
7	16	
Total	100	

(1) (10 points) State whether the following are **TRUE** or **FALSE**. (You do not need to explain you answers.)

(a) If A, B and C are sets then $A \cup (B \cap C) \subseteq (A \cup B) \cap C$.

(b) If A, B and C are sets then $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

(c) $\emptyset \subseteq \{\emptyset\}$

(d) $\emptyset \in \{\emptyset\}$

(e) $1 \to \mathbf{Z}$ is a proposition.

(f) $x \in \mathbf{Z}$ is a proposition.

(g) $(\neg p \to \mathbf{F}) \to p$ is a tautology.

(h) The negation of a tautology is a contradiction.

(i) If $1 + 1 = 3$ then $1 + 1 = 2$.

(j) If $1 + 1 = 2$ then $1 + 1 = 3$.

(2) (10 points) Let A and B be (non-empty) sets and let $f : A \to B$ be a function. Determine whether each of the following propositions imply that f is surjective (indicate with **S**) or that there is not enough information to decide that f is surjective (indicate with **NEI**). (You do not need to explain your answers.)

(a) $\forall a \in A \exists b \in B (f(a) = b)$

(b) $\exists b \in B \forall a \in A (f(a) = b)$

(c) $\exists a \in A \exists b \in B (f(a) = b)$

(d) $\forall b \in B \exists a \in A (f(a) = b)$

(e) $\neg(\exists b \in B \forall a \in A (f(a) \neq b))$

(3) (16 points) Sketch the graphs of the following functions

(a) $f : \mathbf{R} \to \mathbf{Z}$ where $f(x) = -\lfloor x \rfloor$.

(b) $g : \mathbf{N} \to \mathbf{R}$ where $g(x) = -x$.

(4) (16 points) Let A and B be subsets of a universal set U.

(a) Prove carefully that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. (It is not enough to just use Venn diagrams or state that it is an identity.)

(b) Find two sets A and B such that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$. Explain.

(5) (16 points) Recall that the open interval (a, b) is the set of all real numbers x such that $a < x < b$.

(a) Find a function $f : (0, 1) \to (1, \infty)$ which is bijective. Explain

(b) Find a function $f : (0, 1) \to (1, \infty)$ which is injective but not surjective. Explain.

(6) (16 points) Let $P(x, y) : x < y$ be the propositional function with all positive integers as its universe of discourse. Determine whether the following propositions are **TRUE** or **FALSE** and explain your answers carefully.

(a) $\forall x \exists y P(x, y)$

(b) $\forall y \exists x P(x, y)$

(7) (16 points) There are two kinds of inhabitant on an island: Knights, who always tell the truth, and Knaves, who always lie. There is no way to distinguish between knights and knaves visually. You are on the island and have the following encounters with the native inhabitants.

(a) You meet two people, A and B. A says "Neither of us are knights" and B says nothing. Can you determine what A and B are, and if not, can you draw any conclusions?

(b) You then meet two more people, C and D. C says "At least one of us is a knave" and D says nothing. Can you determine what C and D are, and if not, can you draw any conclusions?