A Brief Guide to Writing Proofs

A mathematical proof is a sequence of logical statements which verify the truth of a statement. In this guide, you will find hints and suggestions for how to write a good mathematical proof. Most of it applies to any math course, not just MA2312 and MA2322.

General Comments:

(1) A good proof should lead the reader through your logical arguments, with no assumption that the reader already knows the solution. Using correct English grammar and punctuation will make your logical arguments easier to comprehend for the person reading them.

(2) Always write down the statement you are going to prove, including any assumptions. When you write your proof clearly indicate where it begins and ends.

(3) Keep your reader informed. State what kind of proof you are going to use, indicate when you use assumptions and justify each step of your argument. When in doubt, provide more information provided it is relevant.

(4) Use mathematical symbols correctly. All mathematical symbols and statements have an English translation, you can check whether they make sense by reading your work back to yourself.

Methods of Proof: There are many different types of mathematical proof, you are referred to your text book and the section "Methods of Proof" for more details. Here we will just concentrate on a few types of proof which are probably the most important in the Discrete Math courses.

(1) **Implications** You will frequently have to prove statements of the form $p \to q$ or "if p, then q" in English. Recall that the proposition $p \to q$ is true UNLESS p is true and q is false, so you only need to show that q is true if p is true. Here are two different ways to prove an implication.

   (a) Direct Proof. In a direct proof assume that p is true, and prove that q is true.

   (b) Indirect Proof. Recall that $p \to q$ is logically equivalent to the contrapositive $\neg q \to \neg p$, so another way to prove the implication $p \to q$ is to assume that q is false and show that p is false.

(2) **"If and only if" Statements** Recall that the biconditional $p \leftrightarrow q$ is logically equivalent to $(p \to q) \wedge (q \to p)$. This means that to prove a statement of the form "p if and only if q" or $p \Leftrightarrow q$, you need to prove TWO implications, "if p, then q" and "if q, then p".

(3) **Proofs involving quantifiers** To prove that the proposition $\forall x P(x)$ is true you must find some way to prove that the proposition $P(c)$ is true whatever value c is subsituted for x. However to prove that $\forall x P(x)$ is false, you just need to find one example of a value d of x such that $P(d)$ is false.

   To prove that the proposition $\exists x P(x)$ is true, you just need to find one example c such that $P(c)$ is true. To prove that $\exists x P(x)$ is false, you need to show that $P(d)$ is false for every possible value d. Note that it often makes proofs easier to follow if you use a letter such as c or d to denote a specific value of the variable x.

   Remember also that $\neg \forall x P(x) \Leftrightarrow \exists x \neg P(x)$ and $\neg \exists x P(x) \Leftrightarrow \forall x \neg P(x)$.

(4) **Proof by Contradiction** To prove that a proposition p is true, you may assume the *negation* of p and logically derive a contradiction. This is a valid method of proof because the proposition $\neg p \rightarrow \mathbf{F}$ is true if and only if $\neg p$ is false (check the truth table), which is logically equivalent to p being true.

REMARK!! To prove that the proposition p is true, you should NEVER, EVER assume that p is true. This may sound obvious, but in practice it is very tempting to write down the conclusion within your proof to show "where you are heading". This is not only bad proof writing technique, it may well also lead to logical errors and a totally incorrect conclusion.

If you are writing a solution you should have first written down the statement p which you are trying to prove, and then started the proof. The statement p should NOT appear anywhere within your proof until the very LAST line, when its truth has been established.