Polytechnic University

MA 2322 FINAL MAY 9TH, 2003

Print Name:

Signature:

ID #:

Instructor/Section: Cornick

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You will **NOT** receive full credit for a correct answer without explanation. No calculators.

Problem	Possible	Points
1	20	
2	16	
3	16	
4	16	
5	16	
6	16	
Total	100	

(1) (20 points) State whether the following are **TRUE** or **FALSE**. (No explanation required.)

(a) The cycle C_3 is a bipartite graph.

(b) C_4 is a bipartite graph.

(c) The complete graph K_4 is a bipartite graph.

(d) The 3-cube Q_3 is a bipartite graph.

(e) K_3 has an Euler circuit (a simple circuit which uses every edge.)

(f) K_4 has an Euler circuit.

(g) Q_3 has an Euler circuit.

(h) The complete bipartite graph $K_{3,3}$ is a planar graph.

(i) Q_3 is a planar graph.

(j) $K_{2,4}$ is a planar graph.

(2) (16 points) Let $G = (V, E)$ be a simple graph. G is said to be **self complementary** if G is isomorphic to its complementary graph $\overline{G}$. Recall that two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.

(a) Find a self complementary graph with 4 vertices. Explain. (Hint: Count edges)

(b) Prove that there is no self complementary graph G with 6 vertices. (Hint: What is $G \cup \overline{G}$?)

(3) (16 points) The set S is defined recursively by

$$\begin{array}{ll} 1. & 1 \in S \\ 2. & x, y \in S \Rightarrow x + 2y \in S. \end{array}$$

Prove that S is the set of positive, odd integers.

(4) (16 points) Let $A = \{1, 2, 3, 4\}$.

(a) How many equivalence relations are there on A? Explain.

(b) How many equivalence relations R are there on A with the property that $(1, 2) \in R$. Explain.

(5) (16 points) If $f : A \to A$ is a function, then $G_f = \{(a, f(a)) | a \in A\}$ is a subset of $A \times A$. Therefore G_f defines a relation on A, and consequently an associated directed graph.

(a) Suppose that f is a **bijective** function. What property do the vertices and edges of the associated directed graph have? Explain.

(b) Suppose that G_f defines a reflexive relation on A. What property must the function f have? Explain.

(6) (16 points) A deck of cards contains 52 cards, which are split into 4 **suits** (hearts, clubs, diamonds, spades). Within each suit there are 13 **denominations**

$$2, 3, \ldots 10, J(= 11), Q(= 12), K(= 13), A(= 1 \text{ or } 14).$$

A poker hand consists of 5 cards, and it does **not** matter which order you receive the cards, you may rearrange them in any way to make your best hand. A **straight** in poker consists of 5 cards which you can put in sequence where the cards may or may not be from different suits. For example $3, 4, 5, 6, 7, 8$ and $8, 9, 10, J, Q$ and $A, 2, 3, 4, 5$ and $10, J, Q, K, A$ are straights but $Q, K, A, 2, 3$ is not a straight.

(a) How many different poker hands are there?

(b) How many poker hands are full houses? (A **full house** is 3 cards of one denomination, and 2 cards of another denomination. For example $2, 2, 2, J, J$.)

(c) How many poker hands are flushes? (A **flush** is 5 cards which are all from the same suit but which is not a straight. For example $2, 4, 7, 10, K$ all from the hearts suit.)