

Polytechnic University

MA 2322

FINAL

JULY 14, 2003

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You will **NOT** receive full credit for a correct answer without explanation.

Problem	Possible	Points
1	15	
2	15	
3	10	
4	15	
5	15	
6	15	
7	15	
Total	100	

(1) (15 points) Sketch an example of each of the following **undirected, simple** graphs. No explanation required.

(a) A bipartite graph with 5 vertices and 5 edges.

(b) A tree with 7 edges, such that every vertex has odd degree.

(c) A graph with 6 edges, such that every vertex has odd degree.

(d) A graph such that every vertex has degree 3.

(e) The 3-cube Q_3 .

(2) (15 points) Let $S = \{1, 2, 3\}$. How many relations are there on S which have the following properties. No explanation required. (Hint: Matrices)

(a) Symmetric.

(b) Reflexive.

(c) Antisymmetric.

(d) Symmetric and Reflexive.

(e) Antisymmetric, Symmetric and Reflexive.

- (3) (10 points) Prove that the relation $R = \{(a, b) \mid a \equiv b \pmod{3}\}$ is an equivalence relation on the set of integers.

- (4) (15 points) A simple graph is called **regular** if every vertex of the graph has the same degree, and a regular graph is **n-regular** if every vertex has degree n .
- (a) For which values of m and n is the complete bipartite graph $K_{m,n}$ regular? Explain.

- (b) How many vertices does a 4-regular graph with 10 edges have?

- (5) (15 points) Let A be a finite set. Prove that A has an equal number of subsets with even cardinality as it has subsets with odd cardinality. (Hint: binomial theorem.)

- (6) (15 points) A relation R on a set A is **irreflexive** if for every $x \in A$, $(x, x) \notin R$.
- (a) What property must the directed graph of an irreflexive relation R have? Explain.
- (b) Give an example of a relation R on the set $A = \{a, b, c, d\}$ which is irreflexive and transitive.

- (7) (15 points) A deck of cards contains 52 cards, which are split into 4 **suits** (hearts, clubs, diamonds, spades). Within each suit there are 13 **denominations**
2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace.

In the game of Bridge, the 52 cards are dealt to 4 players so that each player receives 13 cards.

- (a) How many different ways are there to deal the cards to the 4 players. Explain.

- (b) How many different ways are there to deal the cards, so that each player has exactly one Ace. Explain.