

Polytechnic University

MA 2322

MIDTERM

APRIL 15TH, 2003

Print Name:

Signature:

ID #:

Instructor/Section: Cornick

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You will **NOT** receive full credit for a correct answer without explanation. No calculators.

Problem	Possible	Points
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
Total	100	

(1) (10 points) No explanation required in this question. How many integers between 1 and 100 (including 1 and 100)

(a) are divisible by 5 and by 7.

(b) are divisible by 5 or by 7.

(c) are not divisible by 5 and not divisible by 7.

(d) are divisible by 5 but not by 7.

(e) are divisible by 7 but not by 5.

(2) (15 points) The fibonacci numbers are defined recursively by

$$f_0 = 0, f_1 = 1 \text{ and } f_n = f_{n-1} + f_{n-2}$$

Prove that for every $n \geq 1$

$$f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$$

(3) (15 points) Let $A = \{n^2 \mid n \in \mathbf{N}\} = \{0, 1, 4, 9, \dots\}$.

(a) Write down (what you think/hope is) a recursive definition of the set A and call this set S .

(b) Prove that $S = A$.

- (4) (15 points) How many case-insensitive alphanumeric strings are there of length 10 which
- (a) start with three letters and end with two digits. Explain.

(b) start with three letters or end with two digits. Explain.

(5) (15 points) In this problem we count how many paths there are from the point $(0, 0)$ to the point $(3, 3)$ in the xy -plane.

(a) How many paths are there if a path is formed by taking steps of one unit in the positive x direction or positive y direction. Explain.

(b) How many paths of length 10 are there which start at $(0, 0)$ and end at $(3, 3)$ if you are also allowed to take steps of one unit in the *negative* y direction. Explain. (Hint: Consider the expansion of $(x + y + \frac{1}{y})^{10}$.)

- (6) (15 points) How many positive integers with *exactly* four digits have the property that the sum of their digits is 19? Explain. (Hint: Find the solutions of $d_1 + d_2 + d_3 + d_4 = 19$ with suitable restrictions on the d_i .)

- (7) (15 points) How many positive divisors does the number 17,640 have? Explain. (Hint: $17,640 = 2^3 \cdot 3^2 \cdot 5 \cdot 7^2$, try to work out the pattern for much smaller numbers if you get stuck).

- (8) (Extra Credit) Prove that a positive integer n has an odd number of positive divisors if and only if n is a perfect square.