

Polytechnic University

MA 2322

MIDTERM

JULY 3, 2003

Print Name:
Signature:
ID #:
Instructor/Section: Cornick

Directions: You have **90 minutes** to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You will **NOT** receive full credit for a correct answer without explanation. No calculators.

Problem	Possible	Points
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
Total	100	

(1) (10 points) No explanation required in this question. How many case-insensitive strings of three English letters are there

(a) that contain no vowels, if letters can be repeated.

(b) that start with a vowel, if letters can be repeated.

(c) that contain at least one vowel, if letters can be repeated.

(d) that contain exactly one vowel, if letters can be repeated.

(e) that contain no vowels, if letters cannot be repeated.

(2) (15 points) The fibonacci numbers are defined recursively by

$$f_0 = 0, f_1 = 1 \text{ and } f_n = f_{n-1} + f_{n-2}$$

Prove that for every $n \geq 1$

$$f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$$

(3) (15 points) Let S be defined recursively by

$$1 \in S$$

$$\text{If } x \in S \text{ and } y \in S, \text{ then } x + 2y \in S$$

and let A be the set of positive odd integers. Prove that $S = A$. (Hint: Recall that one way to show that two sets are equal is to prove that they are subsets of each other.)

- (4) (15 points) A Discrete Math exam consists of ten **True/False** questions of which the correct answers to three of the questions are **True**, and the answers to the other seven questions are **False**. How many ways can the questions be ordered
- if all the questions with **True** answers must appear consecutively. Explain.

- if no two questions with **True** answers can appear consecutively. Explain.

- (5) (15 points) A computer network consists of six computers. Each computer is directly connected to one or more of the other computers. Prove that there are at least two computers in the network which are directly connected to the same number of computers.

(6) (15 points) How many solutions are there to the equation

$$x_1 + x_2 + x_3 = 13$$

where x_1, x_2 and x_3 are non-negative integers. Explain.

(7) (15 points) Find the coefficient of x^9y^5 in the expansion of

$$\left(x + y + \frac{1}{y}\right)^{22}.$$

Explain your reasoning carefully.