Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. The last page contains formulas that you might find useful. You may tear that page out. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do not want to have Problem 6 graded, you MUST put an “X” in the Points section of Problem 6.

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(1) (Page 86, Problem 20) Suppose that your uncle Gerald has just opened a store that sells television sets. Let $P(x)$ represent the profit (in dollars) that uncle Gerald makes by selling $x$ television sets per week. Suppose that uncle Gerald currently sells $k$ television sets per week. What do the following expressions represent in practical terms?

(a) $P(k) + 600$

(b) $P(k + 600)$

(c) $P^{-1}(600)$

(d) Write an equation that describes the following statement: If uncle Gerald sells 200 fewer television sets per week, then his profit will decrease by 25%.
(2) (Worksheet IV and Appendix H) Use algebra to find the domain of the function \( S(x) \).

\[
S(x) = \sqrt{\frac{3(x + 2)^2 - 6x(x + 2)}{(x + 2)^4}}
\]

Which one of the following intervals represents the domain of \( S \). Circle only ONE interval. You must show all of your work.

(a) \((-\infty, \infty)\)
(b) \((-\infty, -2)\)
(c) \((2, \infty)\)
(d) \((-\infty, -2) \text{ or } (2, \infty)\)
(e) \((-2, 2]\)
(f) \((-2, 2)\)
(g) \((-2, 0) \text{ or } (0, 2)\)
(h) \([-2, 2]\)
(i) \([-2, 2)\)
(3) (Page 146, Problem 19) Scientists observing owl and hawk populations collect the following data. Their initial count for the owl population is 245 owls, and the population grows at the continuous rate of 2.95\% per year. They initially observe 75 hawks and the population doubles every 7 years. Will the hawk population ever be twice as large as the owl population? If so, give the number of years it takes for this to occur.
(4) (Page 347, Problem 33) Find a formula for $f^{-1}(w)$ if

$$f(x) = \frac{2 - \frac{1}{x}}{3 - \frac{2}{x}}.$$ 

Assume that $f$ is defined on a domain where it is invertible.
(5) (Worksheet II) Find all of the exact solutions to the equation in the interval \([-\frac{p}{2}, \frac{p}{2}]\).

\[-6 \cos\left(\frac{3\pi \theta}{p}\right) + 5 = 2\]
(6) (Page 305, Problem 26) Let $y$ be the side opposite to the angle $\theta$ in a right triangle whose hypotenuse is 2. See figure below. Find a formula in terms of $y$ for each of the following expressions.

(a) $\cos(2\theta)$

(b) $\sin^2(\cos^{-1}(y/2))$
During this time interval, which particle(s)

(a) has the least initial velocity?

(b) has the greatest initial velocity?

(c) has the greatest average velocity?

(d) has positive acceleration for the entire period?

(e) has zero acceleration for some period?
(8) (Page 84, Problems 23–32) For each of the functions (a)–(d), find a function from (I)–(VIII) which could be its derivative.

Functions

(a)  
(b)  
(c)  
(d)  

Derivatives

(I)  
(II)  
(III)  
(IV)  
(V)  
(VI)  
(VII)  
(VIII)  

- The derivative of function (a) is:
- The derivative of function (b) is:
- The derivative of function (c) is:
- The derivative of function (d) is:
(9) (Worksheet 1) For what value(s) of $a$ is the function

$$f(x) = \begin{cases} 
-6 & \text{if } x < a \\
\ln(x)\left(1 - \ln(x)\right) & \text{if } x \geq a 
\end{cases}$$

continuous at $x = a$? Show all your work.
(10) (Page 94, Problem 16) A function $f$ satisfies the following conditions:

$f(5) = 20, \quad f'(5) = 2, \quad \text{and} \quad f''(x) < 0 \quad \text{for} \quad -1 \leq x \leq 8.$

(a) Which of the following are possible values for $f(1)$?

(i) 11
(ii) 13
(iii) 12

(b) Use the tangent line approximation to estimate the value of $f(5.3)$. Show your work.
Formulas you might find useful

- The derivative of a function
  \[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

- \[ \frac{d}{dx} x^n = nx^{n-1} \]
- \[ \frac{d}{dx} e^x = e^x \]

For a triangle with sides \(a, b, c\) and angles \(A, B, C\) opposite these sides, respectively.

- Law of Sines:
  \[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \]

- Law of Cosines:
  \[ c^2 = a^2 + b^2 - 2ab \cos C \]

- Double angle:
  \[ \sin(2t) = 2 \sin t \cos t \quad \cos(2t) = \cos^2 t - \sin^2 t \]

- Sum/Difference:
  \[ \sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) \]
  \[ \sin(\theta - \phi) = \sin(\theta) \cos(\phi) - \sin(\phi) \cos(\theta) \]
  \[ \cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi) \]
  \[ \cos(\theta - \phi) = \cos(\theta) \cos(\phi) + \sin(\theta) \sin(\phi) \]