Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. The last page contains formulas that you might find useful. You may tear that page out. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do not want to have Problem 6 graded, you MUST put an “X” in the Points section of Problem 6.

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<th>Problem</th>
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Total 100
(1) (Chapter 6 and Chapter 9 Concepts) Determine whether each of the following statements is TRUE or FALSE. (You do not have to explain.)

(a) If \( \sin(t) = \sin(w) \), then \( t = w \).

(b) If the numerator of a rational function is a polynomial of degree 3, then the rational function has a zero.

(c) If \( f(x) \cdot g(x) \) is an odd function, then both \( f(x) \) and \( g(x) \) are odd functions.

(d) The sum of two even functions is also an even function.

(e) If \( p \) is a polynomial function of degree \( n \), where \( n \) is a positive even integer, then \( p \) is an even function.
(2) Circle the ONE alternative that best completes the sentence. You do not need to explain.

(a) (Page 384, Problem 21) If \( p \) is a polynomial of degree \( n \), where \( n \) is a positive odd integer, then

- (i) \( p \) has an inverse.
- (ii) \( p \) is an odd function.
- (iii) \( p(x) = 0 \) has a solution.
- (iv) \( p(x) \to +\infty \) as \( x \to +\infty \), and \( p(x) \to -\infty \) as \( x \to -\infty \).

(b) (Page 384, Problem 18) If \( f(x) \) is a polynomial of degree \( n \), and \( g(x) \) is a polynomial of degree \( m \), then \( f(g(x)) \) is a polynomial of degree

- (i) \( n \)
- (ii) \( m \)
- (iii) \( n + m \)
- (iv) \( nm \)

(c) (Page 372, Problem 15) Consider power function \( q(x) = kx^{p/3} \), where \( p \) is any positive integer, then

- (i) the domain of \( q(x) \) is \((0, \infty)\).
- (ii) \( q(x) \to 0 \) as \( x \to +\infty \).
- (iii) if \( p \) is an odd integer, then \( q(x) \) is an odd function.
- (iv) \( q(x) \) is concave up.

(d) (Page 376, Problem 4) Let \( u(t) = t^3 \), \( v(t) = 3t \), \( w(t) = t^6 \), and \( z(t) = \log t \). The function which approaches infinity the fastest is

- (i) \( u(t) \)
- (ii) \( v(t) \)
- (iii) \( w(t) \)
- (iv) \( z(t) \)
(3) (Worksheet 4; Page 281, Problem 33) Find the exact value of each of the expressions. You must show all your work.

(a) $\cos \left( \sin^{-1} \left( \frac{1}{4} \right) \right)$  
Hint: Let $\theta = \sin^{-1} \left( \frac{1}{4} \right)$ where $0 \leq \theta \leq \pi/2$.

(b) $\tan \left( \cos^{-1} \left( -\frac{1}{3} \right) \right)$  
Hint: Let $\theta = \cos^{-1} \left( -\frac{1}{3} \right)$ where $\pi/2 \leq \theta \leq \pi$. 
(4) (Worksheet 4; Page 280, Problem 15) Find all the exact solutions for $x$ in the interval $[0, \pi)$. You must show all of your work.

$$1 + \sin(3\theta) = \cos^2(3\theta)$$
(5) (Page 377, Problem 17) Find a possible formula for $f$ assuming $f(1) = 16, \quad f(2) = 128, \quad \text{and}$

(a) $f$ is an exponential function. You must simplify your answer and show all your work.

(b) $f$ is a power function. You must simplify your answer and show all your work.
Let $f(x) = x^3 + 27$, $g(x) = x^2 + 9$, and $h(x) = 2x - 6$. Match each of the functions (a)–(e) to one of the descriptions (I)–(VIII). Note that some functions may match none of the given descriptions. If no description matches, write "none". You do not have to show your work.

(a) $y = \frac{(h(x))^2}{g(x)}$ 
(b) $y = \frac{f(x)}{g(x)h(x)}$

c) $y = \frac{g(x)}{(h(x))^2}$
(d) $y = \frac{g(x)}{h(x)}$

(e) $y = f(x) \cdot g(x)$

(I) Two zeros at $x = \pm 3$, no vertical asymptotes, no horizontal asymptotes.
(II) No zeroes, vertical asymptote at $x = 3$, a horizontal asymptote at $y = 1/2$.
(III) One zero at $x = -3$, no vertical asymptotes, no horizontal asymptotes.
(IV) No zeros, a vertical asymptote at $x = 3$, no horizontal asymptotes.
(V) One zero at $x = -3$, a vertical asymptote at $x = 3$, a horizontal asymptote at $y = 1/2$.
(VI) Two zeros, vertical asymptote at $x = 0$, horizontal asymptote at $y = -4$.
(VII) One zero at $x = 3$, no vertical asymptotes, a horizontal asymptote at $y = 2$.
(VIII) One zero at $x = 3$, no vertical asymptotes, a horizontal asymptote at $y = 4$.

Function (a) is represented by the description _____________.
Function (b) is represented by the description _____________.
Function (c) is represented by the description _____________.
Function (d) is represented by the description _____________.
Function (e) is represented by the description _____________.
(7) (Page 390, Problem 11) Find a possible formula for the polynomial graphed below. You must show your work.
(8) (Page 407, Problem 26) Find a possible formula for the rational function graphed below. You must show your work.
(9) (Worksheet V; Page 281, Problems 21–22) A weight suspended from a spring is vibrating vertically with up being the positive direction. The function

\[ s(t) = 10 \sin \left( \frac{3\pi t}{4} - \frac{\pi}{4} \right) \]

represents the distance in centimeters of the weight from its rest position as a function of time \( t \), where \( t \) is measured in seconds. Find the smallest positive value of \( t \) for which the displacement of the weight above its resting point is 5 cm. Round the answer to three decimal places, if necessary. You must show all of your work.
(10) (Page 372, Problem 16) A person’s weight, \( w \), on a planet of radius \( d \) is given by the formula

\[
    w = kd^{-2}, \quad k > 0,
\]

where the constant \( k \) depends on the masses of the person and the planet; it does not depend on \( d \). (Note that there is a distinction being made here between mass and weight. For example, an astronaut in orbit may be weightless, but he still has mass.)

(a) A man weighs 180 lb on the surface of Earth. How much does he weigh on the surface of a planet as massive as the Earth, but whose radius is three times as large? Show all your work.

(b) What fraction of the Earth’s radius must an equally massive planet have if the surface-weight of the man in part (a) is 1000 lb? Show all your work.
Useful formulas

For a triangle with sides $a$, $b$, $c$ and angles $A$, $B$, $C$ opposite these sides, respectively.

- **Law of Sines:**
  \[
  \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
  \]

- **Law of Cosines:**
  \[
  c^2 = a^2 + b^2 - 2ab \cos C
  \]

- **Double angle:**
  \[
  \sin(2t) = 2 \sin t \cos t \quad \cos(2t) = \cos^2 t - \sin^2 t
  \]

- **Sum/Difference:**
  \[
  \sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) \\
  \sin(\theta - \phi) = \sin(\theta) \cos(\phi) - \sin(\phi) \cos(\theta) \\
  \cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi) \\
  \cos(\theta - \phi) = \cos(\theta) \cos(\phi) + \sin(\theta) \sin(\phi)
  \]

- **Vertex form of a quadratic function:**
  \[
  y = a(x - h)^2 + k
  \]