Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. The last page contains formulas that you might find useful. You may tear that page out. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do not want to have Problem 6 graded, you MUST put an “X” in the Points section of Problem 6.

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(1) (Concepts: Chapter 6 and Chapter 9) Determine whether each of the following statements is True or False. You do not have to explain.

(a) The graph of a rational function cannot have both a vertical and a horizontal asymptote.

(b) The graph of a rational function cannot have three horizontal asymptotes.

(c) The function \( R(x) = \frac{3}{\sqrt{x} - 3} \) is a rational function.

(d) \( \arcsin(\sin(x)) = x \) for all \(-\pi/2 \leq x \leq \pi/2\).

(e) \( \frac{\sin(A)}{\cos(B)} = \tan\left(\frac{A}{B}\right) \).
(2) (Worksheet 4; Page 282, Problem 35) Let \( t \) be a constant with \( \pi/2 < t < \pi \). Find the value of each of the expressions in terms of \( t \), or write DNE if the expression is not defined.

(a) \( \arcsin(\sin(t)) = \) ________________.

(b) \( \sin(\arcsin(t)) = \) ________________.

(c) \( \tan(\arccos(t/\pi)) = \) ________________. Show your work.
   
   Hint: Let \( \theta = \arccos(t/\pi) \).

(d) \( \cos(2 \arccos(t/8)) = \) ________________. Show your work.
(3) (Page 280, Problem 14) Find all the exact solutions for $x$ in the interval $[0, 5\pi]$. You must show all of your work.

$$2 \cos\left(\frac{1}{2} \theta\right) \sin\left(\frac{1}{2} \theta\right) = \cos\left(\frac{1}{2} \theta\right).$$
(4) (Worksheet 4; Practice Exam) A water wheel is 10 meters in diameter and makes one revolution every 4 minutes. The water level is 4.5 meters below the center of the wheel. You put a mark on the perimeter of the wheel. In each revolution, for how many minutes is your mark above the water? Show all your work.
(5) (Page 384, Problems 18 and 21) Determine whether each of the following statements is True or False. You do not have to explain.

(a) If $f(x)$ and $g(x)$ are two polynomials of degree 5, then the composition $f(g(x))$ is a polynomial of degree 10.

(b) If $f(x)$ and $g(x)$ are two polynomials of degree 5, then $f(x)g(x)$ is a polynomial of degree 10.

(c) If $p(x)$ is an odd degree polynomial, then $p(x)$ is an odd function.

(d) If $p(x)$ is an odd degree polynomial, then $p(x)$ is an invertible function.

(e) If $p(x)$ is an odd degree polynomial and if $p(x) \to +\infty$ as $x \to +\infty$, then $p(x) \to -\infty$ as $x \to -\infty$. 
(6) (Page 377, Problem 19) The table below gives values for three function, $f$, $g$, and $h$. One is exponential, one is a power function, and one is a linear function.

<table>
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<tr>
<th>$x$</th>
<th>$-2$</th>
<th>0</th>
<th>3</th>
<th>6</th>
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<tbody>
<tr>
<td>$f(x)$</td>
<td>16</td>
<td>0</td>
<td>36</td>
<td>144</td>
</tr>
<tr>
<td>$g(x)$</td>
<td>1/3</td>
<td>3</td>
<td>81</td>
<td>2187</td>
</tr>
<tr>
<td>$h(x)$</td>
<td>-35</td>
<td>5</td>
<td>65</td>
<td>125</td>
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</table>

(a) Identify the exponential function and find a possible formula for it. Show all your work.

(b) Identify the power function and find a possible formula for it. Show all your work.
(7) (Page 405, Problem 6) Let \( f(x) = x^2 + 5x + 6 \) and \( g(x) = x^2 + 9 \). For each of the following descriptions, select from the functions I–V the one/ones which satisfies/satisfy it. There may be more than one function for each description, or none at all. You do not have to show your work.

(a) No \( x \)-intercept and two vertical asymptotes. ANSWER: ________________

(b) No horizontal asymptote. ANSWER: ________________

(c) Two zeros and no vertical asymptote. ANSWER: ________________

(d) Vertical asymptote at \( x = 0 \). ANSWER: ________________

(e) Horizontal asymptote at \( y = 0 \). ANSWER: ________________

I. \( y = \frac{f(x)}{g(x)} \)

II. \( y = \frac{g(x)}{f(x)} \)

III. \( y = g\left( -\frac{1}{x} \right) \)

IV. \( y = \frac{g(x)}{f(x)} - 1 \)

V. \( y = f(x)g(x) \)
The rational function
\[ C(x) = \frac{130x}{100 - x}, \quad 0 \leq x < 100, \]
describes the cost, \( C(x) \), in millions of dollars, to inoculate \( x\% \) of the population against a particular strain of flu.

(a) Find and interpret \( C(20) \).

(b) What is the equation of the vertical asymptote?

c) What does the vertical asymptote mean in terms of the problem?

d) Evaluate \( C^{-1}(10) \).

e) Interpret \( C^{-1}(10) \) in the context of this problem.
(9) (Page 372, Problems 13–15) In each of the following parts, circle the **ONE** alternative that best completes the sentence. You do not need to explain.

(a) If \( f(x) = 3x^4 \) and \( g(x) = 2x^3 \), then

(i) \( g(f(x)) = 48x^{12} \).
(ii) \( g(f(x)) = 6x^7 \).
(iii) \( g(f(x)) = 54x^{12} \).
(iv) \( g(f(x)) = 48x^7 \).
(v) None of the above.

(b) If \( t(x) = kx^{p/2} \) where \( p \) is any positive integer, then

(i) the range of \( t(x) \) is \([0, \infty)\).
(ii) \( t(x) \) has no horizontal asymptote.
(iii) \( t(x) \) is defined for all real value of \( x \).
(iv) None of the above.

(c) If \( k^{-1}(t) = j(t) \), \( k(3) = 5 \), and \( k(7) = 3 \), then \( k(j(9)) - k(3)j(3) = \)

(i) 6.
(ii) 0.
(iii) \(-30\).
(iv) is not defined.
(v) None of the above.

(d) If \( q(x) = x^p \), where \( p \) is a positive odd integer, then

(i) \( q(x) \leq q(q(x)) \) for all positive values of \( x \).
(ii) \( q(x) \) is an odd function.
(iii) Both of the above.
(iv) None of these.
(10) (Page 390, Problem 17) Find a possible formula for the polynomial graphed below. You must show all of your work.
(Hint: Observe the vertical shift.)
Useful formulas
For a triangle with sides $a$, $b$, $c$ and angles $A$, $B$, $C$ opposite these sides, respectively.

- **Law of Sines:**
  \[
  \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
  \]

- **Law of Cosines:**
  \[
  c^2 = a^2 + b^2 - 2ab \cos C
  \]

- **Double angle:**
  \[
  \sin(2t) = 2 \sin t \cos t \quad \cos(2t) = \cos^2 t - \sin^2 t
  \]

- **Sum/Difference:**
  \[
  \sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta)
  \]
  \[
  \sin(\theta - \phi) = \sin(\theta) \cos(\phi) - \sin(\phi) \cos(\theta)
  \]
  \[
  \cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi)
  \]
  \[
  \cos(\theta - \phi) = \cos(\theta) \cos(\phi) + \sin(\theta) \sin(\phi)
  \]

- **Vertex form of a quadratic function:**
  \[
  y = a(x - h)^2 + k
  \]