SECOND FUNDAMENTAL THEOREM OF CALCULUS

Consider the function \( f(x) = \sin(x^2) \). We would like to find an antiderivative, \( F \), of the function \( f \). However, there is no way to describe \( F \) in terms of known elementary functions. However, from the graph of \( f \), we can certainly obtain a graph of \( F \). The fundamental Theorem of Calculus gives a way to express antiderivatives of given functions.

For example: \( F(x) = \int_0^x \sin(t^2) \, dt \) is a specific antiderivative of a continuous function \( f \). This particular \( F \) satisfies \( F(0) = 0 \). Note that the independent variable of the function \( F \) is \( x \) which appears as the upper limit of integration. One can use any variable inside the integral sign.

More generally, the function \( F \) described by \( F(x) = \int_a^x f(t) \, dt \) is the antiderivative of \( f \) which satisfies \( F(a) = 0 \).

So what do we do if we want to construct the antiderivative of \( f(x) = e^{-x^2} \) passing through the point \((3, 105)\)? This would be accomplished by the writing

\[
F(x) = \int_3^x e^{-t^2} \, dt + 105
\]

Note that \( F'(x) = e^{-x^2} \) and \( F(3) = \int_3^3 e^{-t^2} \, dt + 105 = 105 \) as required.

Of course, we can now combine \( F \) with other known functions to construct more complex functions. For example if \( F(x) = \int_a^x f(t) \, dt \), and \( h \) is another function, then we can construct the composite function \( G(x) = F(h(x)) = \int_a^{h(x)} f(t) \, dt \). Of course, if we want to differentiate \( G \), we must use the chain rule.