Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last page contains formulas that you might find useful. You may tear that page out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

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(1) (Page 251, Problems 4–9) Suppose that
\[ \int_{2}^{4} f(x) \, dx = 3.7, \quad \int_{2}^{16} f(x) \, dx = 4.1, \quad \text{and} \quad \int_{2}^{4} (f(x))^2 \, dx = 2. \]
Evaluate each of the following integrals. Write “NEI” if there is not enough information. Show all your work.

(a) \[ \int_{2}^{4} (2 + f(x))^2 \, dx = \]

(b) \[ \left( \int_{4}^{16} f(x) \, dx \right)^2 = \]
(2) (Page 235, Problem 24)

(a) Estimate $\int_{1}^{3} \sqrt{x} \, dx$ using left-hand sum with four equal subdivisions. Show all your work.

(b) Calculate the exact value of $\int_{1}^{3} \sqrt{x} \, dx$. Show all your work.
(a) Oil leaks out of a tanker at a rate of \( r = f(t) \) gallons per minute, where \( t \) is in minutes. Write a definite integral expressing the total quantity of oil which leaks out of the tanker in the first hour.

(b) If \( n(t) \) is the rate of growth of a child in pounds per year, what does \( \int_{4}^{9} n(t) \, dt \) represent? Include units in your answer.

(c) A honeybee population starts with 150 bees and increases at the rate of \( y(t) \) bees per week. What does \( 150 + \int_{0}^{t} y(s) \, ds \) represent? Include units in your answer.
(4) (Sample Exam) Use geometry and properties of integrals to find the **exact** value of

\[ \int_{-4}^{0} \left( |2x + 3| + 2\sqrt{16 - x^2} \right) \, dx. \]

Show all your work.
(5) (Concepts) Determine whether each of the following statements is True or False. You do not have to explain.

(a) \( f(x) = \ln|5x| + 5 \) is an antiderivative of \( g(x) = \frac{1}{x} \).

(b) \( f(x) = \frac{\sin(2x) + 1}{2} \) is an antiderivative of \( g(x) = \cos(2x) \).

(c) If \( g'(p) = 0 \), then \( g \) must have either a local minimum or a local maximum at \( x = p \).

(d) If \( f'(x) < g'(x) \) for \( 0 < x < 1 \), then \( f(x) < g(x) \) for \( 0 < x < 1 \).

(e) If a differentiable function \( g \) is increasing on the interval \([0, \infty)\), then \( k(x) = x^2g(x) \) must also be increasing on the interval \([0, \infty)\).
(a) (Page 281, Problem 25) \( \frac{d}{dx} \left( \int_{x^2}^{2x} \sin(2x) \, dx \right) = \)

(i) \( 2 \cos(2x) \).

(ii) \( \sin(2x^2) - \sin(2xe^2) \).

(iii) \( 2x \sin(2x^2) - e^2 \sin(2xe^2) \).

(iv) \( 2x \cos(2x) \).

(v) \( 2x \cos(2x^2) - e^2 \cos(2xe^2) \).

(vi) None of the above.

(b) (Page 281, Problem 12) If \( F'(x) = e^{-2x} \) and \( F(0) = 2 \), then \( F(1) = \)

(i) \( 1 + e^{-2} \).

(ii) \( \frac{1 + e^{-2}}{2} \).

(iii) \( \frac{4 - e^{-2}}{2} \).

(iv) \( \frac{5 - e^{-2}}{2} \).

(v) \( \frac{1 - e^{-2}}{2} \).
(7) In each part, circle the correct choice.

(a) (Page 272, Problem 67) \( \int_{1}^{e} \frac{1+y^2}{\sqrt{y^3}} dy = \)

(i) \( \frac{2e^2 + 4\sqrt{e} - 6}{3\sqrt{e}} \).

(ii) \( \frac{e + e^3/3}{2/5 \ e^{5/2}} - \frac{10}{3} \).

(iii) \( -2\sqrt{e} + \frac{2\sqrt{e^3}}{3} + \frac{4}{3} \).

(iv) \( e + \frac{2}{3}e^{3/2} - \frac{5}{3} \).

(v) None of the above.

(b) (Page 271, Problem 18) \( \int (5e^{5t} - t) \ dt = \)

(i) \( e^{5t} - 1 + C \).

(ii) \( 5e^{5t} - \frac{t^2}{2} + C \).

(iii) \( 25e^{5t} - 1 + C \).

(iv) \( e^{5t} - \frac{t^2}{2} + C \).

(v) None of the above.
Let \( f(x) \) be a function. The figure below shows the graph of \( f'(x) \).

Circle the **LARGER** of the two values in each part. You do not have to explain.

(a) \( f(-2) \) or \( f(0) \)

(b) \( f''(-2) \) or \( f''(0) \)

(c) \( \int_{0}^{2} f''(t) \, dt \) or \( \int_{2}^{4} f''(t) \, dt \)

(d) \( \int_{-2}^{0} f'(t) \, dt \) or \( \int_{0}^{2} f'(t) \, dt \)

(e) \( \int_{-4}^{-2} |f(t)| \, dt \) or \( \int_{-1}^{0} f(t) \, dt \)
(9) (Page 272, Problem 81) The average value of the function \( v(x) = \frac{3 + 4x^2}{x^2} \) on the interval \([1, c]\) is equal to 5. Find the value of \( c \). Show all your work.
Formulas you might find useful

• The derivative of a function

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

• Some rules of differentiation

\[ \frac{d}{dx} (cf(x)) = cf'(x) \]
\[ \frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \]
\[ \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \]
\[ \frac{d}{dx} f(g(x)) = f'(g(x))g'(x) \]

• Differentiation formulas

| \( \frac{d}{dx} (x^n) = nx^{n-1} \) | \( \frac{d}{dx} (e^x) = e^x \) | \( \frac{d}{dx} (a^x) = (\ln a)a^x \) |
| \( \frac{d}{dx} (\ln x) = \frac{1}{x} \) | \( \frac{d}{dx} (\sin(x)) = \cos x \) | \( \frac{d}{dx} (\cos(x)) = -\sin x \) |
| \( \frac{d}{dx} (\tan(x)) = \sec^2 x \) | \( \frac{d}{dx} (\sec(x)) = \sec x \tan x \) | \( \frac{d}{dx} (\cot(x)) = -\csc^2 x \) |
| \( \frac{d}{dx} (\arcsin(x)) = \frac{1}{\sqrt{1-x^2}} \) | \( \frac{d}{dx} (\arccos(x)) = -\frac{1}{\sqrt{1-x^2}} \) | \( \frac{d}{dx} (\csc(x)) = -\csc x \cot x \) |
| \( \frac{d}{dx} (\sinh(x)) = \cosh(x) \) | \( \frac{d}{dx} (\cosh(x)) = \sinh(x) \) | \( \frac{d}{dx} (\arctan(x)) = \frac{1}{1+x^2} \) |
| \( \frac{d}{dx} (\tanh(x)) = \frac{1}{\cosh^2(x)} \) |

• The linear approximation of a function \( f \) at \( a \) is given by

\[ y = f(a) + f'(a)(x - a) \]

• Geometry Formulas

Here \( V \) is the volume, \( S \) is the surface area, \( h \) is the height and \( r \) is the radius.

Cylinder with top and bottom: \( V = \pi r^2 h, S = 2\pi rh + 2\pi r^2 \)
Cone: \( V = \frac{1}{3}\pi r^2 h \)
Sphere: \( V = \frac{4}{3}\pi r^3, S = 4\pi r^2 \)