1. Peter with his friends visited the same Café in Wonderland four times last week.

   - The first time they bought 3 cups of coffee, 2 muffins and one bagel, and they paid 20.50 in Wonderlandian dollars.
   - The second time they bought 2 cups of coffee, one tea, one muffin and one bagel, and this cost $16.00.
   - The third time they had 1 cup of tea and one bagel, which was $5.00.
   - The fourth time they had one cup of coffee with 1 muffin and one cup of tea, which cost $10.50.

Set up a system of linear equations for this problem; and use the Gaussian algorithm to find the price of each item.

2. Use the Gauss-Jordan algorithm to find a polynomial \( p(t) \) of degree 3 such that \( p(1) = 1, \ p(2) = 5, \ p'(1) = 2, \) and \( p'(2) = 9, \) where \( p'(t) \) is the derivative of \( p(t). \)

   *Hint:* A polynomial of degree 3 can be written as \( p(t) = at^3 + bt^2 + ct + d. \)

3. Apply the Gaussian algorithm to transform the following complex matrix into row-echelon form

\[
\begin{pmatrix}
2 + i & -1 + 2i & 2 \\
1 + i & -1 + i & 1 \\
1 + 2i & -2 + i & 1 + i
\end{pmatrix}
\]

You have to simplify the entries.
4. Determine the value(s) of $k$ for which the system

\[
\begin{align*}
\begin{aligned}
x + y - z &= k \\
2x + 3y + kz &= 3k \\
x + ky + 3z &= 2k
\end{aligned}
\end{align*}
\]

in unknowns $x$, $y$ and $z$ has

(a) a unique solution,

(b) no solutions,

(c) infinitely many solutions.

5. Use the Gauss-Jordan algorithm to find all solutions of the following system of linear equations in $\mathbb{Z}_3$:

\[
\begin{align*}
\begin{aligned}
x_1 + x_2 + x_3 &= 2 \\
2x_1 + 2x_2 + 2x_3 + x_4 &= 2 \\
x_1 + 2x_2 + 2x_3 &= 1 \\
2x_1 + 2x_2 + x_4 &= 2
\end{aligned}
\end{align*}
\]