If you have the second edition of the MATLAB Manual, then read Chapter 2 and 8 before starting these problems. If you have the third edition of the MATLAB Manual, then read Chapter 3 and 10 before starting these problems.

MATLAB solutions: The MATLAB solutions must be submitted in the form of a printed out MATLAB diary (see the manual or MATLAB help for details) containing explanatory text, input, and output. Text must be used to explain what you are doing and to interpret your results. Your input and MATLAB’s output must be also exhibited. (Uninformative and lengthy MATLAB output should be suppressed.) The MATLAB editor should be used to edit and print the saved diary. If plots are required, they should be printed out separately. However, the MATLAB plot commands must be exhibited in printout of the diary. Manually solved MATLAB problems will not be accepted.

MATLAB downloads: You need to download the following M-files from a website maintained by the first author, John Polking: dfield.m and pplane.m. You must download the proper version of these files, corresponding to the version of MATLAB installed on your laptop. Choose wisely. The files must be downloaded and placed into your MATLAB work directory (typically, this is C:\matlabR12\work), so that they are accessible when needed. These MATLAB files are needed for many of the assigned MATLAB problems from PA.

(1) Consider the differential equation

\[(1 + t^2)y' + 4ty = t.\]

(a) Use dfield6 to calculate and plot a few solutions with different initial points. (Use the display window defined by \(t \in [-5, 5]\) and \(y \in [-5, 5]\).) In particular, plot the solution curve with initial condition \(y(1) = 1/4\) (use Options→Keyboard input). Print out the Figure Window and turn it in as part of this assignment.

(b) What do you conjecture is the limiting behavior of the solutions of this differential equation as \(t \to \infty\)?

(c) Find the general analytic solution to this equation.

(d) Verify the conjecture you made in part (b), or if you no longer believe it, make a new conjecture and verify that.

(2) Verify that \(y = 10 - \frac{t^2}{2}\) is a solution to the differential equation \(yy' + ty = 0\).

**Hint:** Use the symbolic toolbox (syms) and diff. To verify that \(y(t) = ce^{-t^2}\) is a solution of \(y' + 2ty = 0\), where \(c\) is a constant, enter

\[
g >> \text{syms } y \ t \ c \\n\]

\[
g >> y = c \exp(-t^2) \;
\]

to define the symbolic variables and the potential solution. Then

\[
g >> \text{diff}(y,t)+2*t*y \\n\]

\[
g ans= \\n\]

\[
g 0 \\n\]

verifies that you have a solution.
(3) First determine the independent variable, then use `dsolve` to find the general solution to the equation $x(y' - y) = e^x$.

**Hint:** Use `dsolve('x*(Dy-y)= exp(x)','x').`

(4) First determine the independent variable, then use `dsolve` to find the solution to the following initial value problem. Use `ezplot` to plot the solution over the indicated time interval.

$$yy' + ty = 0, \quad y(1) = 4, \quad [-4, 4].$$

(5) Use `dsolve` to obtain the solution of the following second order differential equation and the `simple` command to find the simplest form of the solution. Use `ezplot` to sketch the solution on the indicated time interval.

$$y'' + 16y = 3 \sin(4t), \quad y(0) = 0, y'(0) = 0, \quad [0, 32\pi]$$

(6) Find the solution to the initial value problem

$$y'' = ty' + y + 1, \quad y(0) = 1, y'(0) = 0.$$ 

(7) Suppose we start with a population of 100 individual at time $t = 0$, and that the population is correctly modelled by the logistic equation. Suppose that at time $t = 2$ there are 200 individuals in the population, and that the population reaches steady state with a population of 1000. Plot the population over the interval $[0, 20]$. What is the population at time $t = 10$?