If you have the second edition of the MATLAB Manual, then read Chapter 10 before starting these problems. If you have the third edition of the MATLAB Manual, then read Chapter 12 before starting these problems.

(1) **Repeated Eigenvalue.** Use MATLAB to show that the matrix

\[
A = \begin{pmatrix} -4 & 1 \\ -1 & -2 \end{pmatrix}
\]

have an eigenvalue of algebraic multiplicity two, but geometric multiplicity one. Use the identity \( tA = \lambda tI + t(A - \lambda I) \), where \( \lambda \) is the eigenvalue of the matrix, then \( e^{tA} = e^{\lambda t}(I + t(A - \lambda I)) \) to simplify the exponential matrix \( e^{tA} \).

(2) **Repeated Eigenvalue.** Use MATLAB to show that the system

\[
\mathbf{x}' = \begin{pmatrix} 7 & -1 \\ 1 & 5 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}_0 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}
\]

have an eigenvalue of algebraic multiplicity two, but geometric multiplicity one. Compute the exponential matrix, then compute the solution of the given initial value problem.

(3) **Symbolic Toolbox.** Compute the solution in exercise (2) via the exponential matrix. **Hint:** To solve the IVP

\[
\mathbf{x}' = \begin{pmatrix} -3 & 1 \\ -4 & 1 \end{pmatrix} \mathbf{x}
\]

\[
>> \text{syms } t
\]
\[
>> A=\text{syms}([[-3,1];[-4,1]]);
\]
\[
>> \text{x0}=\text{sym}([1;1]);
\]
\[
>> x=\text{expm}(t*A)*x0
\]