Directions: Complete all questions clearly and neatly. You must show all work to have credit. Unclear work will not be graded. THIS IS A CRUCIAL HOMEWORK UNDERSTAND IT WELL FOR YOUR NEXT EXAM.

<table>
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<th>Problem</th>
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</table>
(1) The joint density of $X$ and $Y$ is given by

$$f(x, y) = \begin{cases} 
xe^{(-x+y)} & x > 0, y > 0 \\
0 & otherwise 
\end{cases}$$

(a) Compute the density of $X$.

(b) Compute the density of $Y$.

(c) Are $X$ and $Y$ independent?
(2) Suppose that $X$ and $Y$ are independent continuous random variables. Show that

(a) $P\{X + Y \leq a\} = \int_{-\infty}^{\infty} F_X(a - y) f_Y(y) \, dy$

where $f_Y$ is the density function of $Y$, and $F_X$ is the distribution function of $X$.

(b) $P\{X \leq Y\} = \int_{-\infty}^{\infty} F_X(y) f_Y(y) \, dy$

where $f_Y$ is the density function of $Y$, and $F_X$ is the distribution function of $X$. 
When a current \( I \) (measured in amperes) flows through a resistance \( R \) (measured in ohms), the power generated (measured in watts) is given by \( W = I^2R \). Suppose that \( I \) and \( R \) are independent random variables with densities

\[
\begin{align*}
f_I(x) &= 6x(1 - x) \quad 0 \leq x \leq 1 \\
f_R(x) &= 2x \quad 0 \leq x \leq 1
\end{align*}
\]

Determine the density function of \( W \).
(4) The density function of $X$ is given by

$$f(x) = \begin{cases} 
    a + bx^2 & 0 \leq x \leq 1 \\
    0 & otherwise
\end{cases}$$

If $E[X] = \frac{3}{5}$, find $a$, $b$. 
(5) Suppose that $X$ is equally likely to take on any of the values 1, 2, 3, 4. Compute
(a) $E[X]$

(b) $Var(X)$
(6) If $X_1$ and $X_2$ have the same probability distribution function, show that
\[ \text{Cov}(X_1 - X_2, X_1 + X_2) = 0 \]
Note that independence is not being assumed.
(7) Suppose that $X$ and $Y$ are independent random variables having the common density function

$$f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & otherwise \end{cases}$$

Find the density function of the random variable $\frac{x}{y}$. 
The joint density function of $X$ and $Y$ is

$$f(x, y) = \begin{cases} 
xy & 0 < x < 1, 0 < y < 2 \\
0 & \text{otherwise}
\end{cases}$$

(a) Are $X$ and $Y$ independent? Explain.

(b) Find the density function of $X$.

(c) Find the density function of $Y$.

(d) Find the joint distribution function.

(e) Find $E[Y]$.

(f) Find $P\{X + Y < 1\}$.