Some Possible Useful Formulas:

(1) For a random sample \(x_1, x_2, \ldots, x_n\), the sample mean and the sample variance are defined as

\[ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \]

and

\[ s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]

(2) Number of ways to choose \(r\) out of \(n\) objects without repetition where order matters:

\[ P(n, r) = n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!} \]

(3) Number of ways to choose \(r\) out of \(n\) objects without repetition where order does not matters:

\[ C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} \]

(4) Number of ways to choose \(r\) out of \(n\) objects with repetition where the order matters:

\[ n^r \]

(5) The conditional probability of \(A\), given \(B\), is:

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) > 0. \]

(6) Bayes’ formula: If the events \(B_1, B_2, \ldots, B_k\) constitute a partition of the sample space \(S\), where \(P(B_i) > 0\) for \(i = 1, 2, \ldots, k\), then for any event \(A\) in \(S\) such that \(P(A) > 0\), \(P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \) for \(r = 1, 2, \ldots, k\)

(7) The cumulative distribution function \(F(x)\) of a random variable \(X\) with p.d.f. \(f(x)\) is \(F(x) = P(X \leq x)\).

(8) The expectation of a discrete random variable \(X\) with values \(x_1, x_2, \ldots, x_n\) is

\[ E(X) = \sum_{i=1}^{n} x_i P(X = x_i) = \sum_{i=1}^{n} x_i f(x_i) \]

(9) The variance of a discrete random variable \(X\) with values \(x_1, x_2, \ldots, x_n\) is

\[ Var(X) = E(X^2) - (E(X))^2 \]

(10) Assume that \(X\) is a continuous random variable with p.d.f. \(f(x)\). The expectation and the variance \(X\) are defined as, \(E(x) = \int_{-\infty}^{\infty} xf(x)dx\) and

\[ Var(X) = E[(X - E(X))^2] = E(x^2) - (E(X))^2 \]

The moment generating function of \(X\), if it exists, is defined as

\[ M(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)dx; \quad -h < t < h \]
(11) Let random variables $X$ and $Y$ have the joint probability mass function $f(x, y)$ with space $S$. The marginal probability mass function of $X$, is defined by

$$f_X(x) = \sum_{y \in S_Y} f(x, y) = P(X = x), \quad x \in S_X$$

The marginal probability mass function of $Y$, is defined by

$$f_Y(y) = \sum_{x \in S_X} f(x, y) = P(Y = y), \quad y \in S_Y$$

(12) The random variables $X$ and $Y$ are independent if and only if

$$f(x, y) = f_X(x)f_Y(y), \quad x \in S_X, \quad y \in S_Y$$

Similarly, you can define the marginal probability density functions for random variables.

(13) For random variables $X$ and $Y$, the covariance of $X$ and $Y$ is defined as

$$Cov(X, Y) = \sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - E(X)E(Y)$$

If the standard deviations $\sigma_X$ and $\sigma_Y$ are positive, then the correlation coefficient is defined as

$$\rho = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

(14) If the random variable follows a normal distribution $N(\mu, \sigma^2)$ then $\frac{X - \mu}{\sigma}$ follows the standard normal distribution, i.e., $N(0, 1)$.

(15) (The Central Limit Theorem) If $\bar{X}$ is the mean of a random sample $X_1, X_2, ..., X_n$ of size $n$ from a distribution with a finite mean $\mu$ and a finite positive variance $\sigma^2$, then the distribution of

$$W = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\sum_{i=1}^{n} X_i - n \mu}{\sqrt{n} \sigma}$$

is $N(0, 1)$ in the limit as $n \to \infty$.

(16) If the random variable $X$ is $N(\mu, \sigma^2), \sigma^2 > 0$, then the random variable $V = \frac{(X - \mu)^2}{\sigma^2}$ is $\chi^2(1)$. 
DISTRIBUTIONS

Bernoulli
\[ f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1 \]
\[ M(t) = 1 - p + pe^t \]
\[ \mu = p, \quad \sigma^2 = p(1-p) \]

Binomial
\[ f(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, \ldots, n \]
\[ M(t) = (1 - p + pe^t)^n \]
\[ \mu = np, \quad \sigma^2 = np(1-p) \]

Poisson
\[ f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots \]
\[ M(t) = e^{\lambda(e^t-1)} \]
\[ \mu = \lambda, \quad \sigma^2 = \lambda \]

Uniform (a, b)
\[ f(x) = \frac{1}{b-a}, \quad a < x < b \]
\[ M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)} \]
\[ \mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12} \]

Exponential (\(\theta\))
\[ f(x) = \frac{1}{\theta}e^{-x/\theta}, \quad 0 \leq x < \infty \]
\[ M(t) = \frac{1}{(1-\theta t)^\alpha} \]
\[ \mu = \alpha\theta, \quad \sigma^2 = \alpha\theta^2 \]

Normal \(N(\mu, \sigma^2)\)
\[ f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \]
\[ M(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}} \]
\[ \text{mean} = \mu, \quad \text{variance} = \sigma^2 \]