Directions: Complete all questions clearly and neatly. You must show all work to have credit. Unclear work will not be graded. THIS IS A CRUCIAL HOMEWORK UNDERSTAND IT WELL FOR YOUR NEXT EXAM.
(1) The moment generating function of $X$ is given by $M_X(t) = \exp\{2e^t - 2\}$ and that of $Y$ by $M_Y(t) = \left(\frac{3}{4} e^t + \frac{1}{4}\right)^{10}$. If $X$ and $Y$ are independent, what are

(a) $P\{X + Y = 2\}$?

(b) $P\{XY = 0\}$?

(c) $E[XY]$?
(2) Let $X_1, X_2, X_3$ be three independent random variables with binomial distributions $b(4, 1/2), b(6, 1/3)$, and $(12, 1/6)$, respectively. Find

(a) $P(X_1 = 2, X_2 = 2, X_3 = 5)$.

(b) $E(X_1X_2X_3)$.

(c) The mean and the variance of $Y = X_1 + X_2 + X_3$. 
(3) Three components are placed in series. The time in hours to failure of each has the p.d.f.

\[ f(x) = \frac{x}{500^2} e^{-x/500}, \quad 0 < x < \infty \]

Since they are in series, we are concerned about the minimum time \( Y \) to failure of the three. Assuming independence, find the c.d.f. and the p.d.f. of \( Y \) and compute \( P(Y \leq 300) \).

HINT: \( G(y) = P(Y \leq y) = 1 - P(Y > y) = 1 - P(all \ three > y) \).
(4) The height of adult women in the United States is normally distributed with mean 64.5 inches and standard deviation 2.4 inches. Find the probability that a randomly chosen woman is

(a) less than 63 inches tall;

(b) less than 70 inches tall;

(c) between 63 and 70 inches tall.

(d) Alice is 72 inches tall. What percentage of women is shorter than Alice?

(e) Find the probability that the average of the heights of two randomly chosen women exceeds 66 inches.
(5) The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years?
(6) Let $X$ have the p.d.f. $f(x) = xe^{-x^2/2}$, $0 < x < \infty$. Find the p.d.f. of $Y = X^2$. 