Directions: Complete all questions clearly and neatly. You must show all work to have credit. Unclear work will not be graded. THIS IS A CRUCIAL HOMEWORK. UNDERSTAND IT WELL FOR YOUR NEXT EXAM.
(1) If $X_1, X_2, ..., X_n$, are independent gamma random variables with parameters $(\alpha, \lambda_1), (\alpha, \lambda_2), ..., (\alpha, \lambda_n)$, respectively, show that $X_1, X_2, ..., X_n$ is a $(\alpha, \lambda_1 + \lambda_2 + ... + \lambda_n)$ gamma random variable.
(2) If $Z_1, Z_2, ..., Z_n$ are independent standard normal random variables, show that $Y = Z_1^2 + Z_2^2 + ... + Z_n^2$ is $X^2(n)$ random variable.
(3) Let $X$ equal the weight of the soap in a 6-pound box. Assume that the distribution of $X$ is $N(6.05, 0.0004)$.

(a) Find $P(X < 6.0171)$.

(b) If nine boxes of soap are selected at random from the production line, find the probability that at most two boxes weigh less than 6.0171 pounds each. HINT: Let $Y$ equal the number of boxes that weigh less than 6.0171 pounds.

(c) Let $\bar{X}$ be the sample mean of the nine boxes. Find $P(\bar{X} \leq 6.035)$. 
(4) The lifetime of a certain electrical part is a random variable with mean 100 hours and standard deviation 20 hours. If 16 such parts are tested, find the probability that the sample mean is

(a) less than 104:

(b) between 98 and 104 hours.
(5) If $X$ is binomial with parameters $n = 150$, $p = 0.6$, compute the exact value of $P\{X \leq 80\}$ and compare with its normal approximation both

(a) making use of the continuity correction and

(b) not making use of the continuity correction
(6) Let $Y = X_1 + X_2 + \ldots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose p.d.f. is $f(x) = (3/2)x^2$, $-1 < x < 1$. Using the p.d.f. of $Y$, we find that $P(-0.3 \leq Y \leq 1.5) = 0.22788$. Use the central limit theorem to approximate this probability.
(7) Let $X$ equal the weight in grams of a miniature candy bar. Assume that $\mu = E(X) = 24.43$ and $\sigma^2 = \text{Var}(X) = 2.20$. Let $\overline{X}$ be the sample mean of a random sample of $n = 30$ candy bars. Find

(a) $E(\overline{X})$.

(b) $\text{Var}(\overline{X})$.

(c) $P(24.17 \leq \overline{X} \leq 24.82)$, approximately.
(8) Let $X$ equal the number out of $n = 48$ mature aster seeds that will germinate when $p = 0.75$ is the probability that a particular seed germinates. Approximate $P(35 \leq X \leq 40)$. 
(9) If $X$ is $b(100, 0.1)$, find the approximate value of $P(12 \leq X \leq 14)$, using 
(a) The normal approximation. 

(b) The Poisson approximation. 

(c) The binomial.