**Directions:** Complete all questions clearly and neatly. You must show all work to have credit. Unclear work will not be graded. THIS IS A CRUCIAL HOMEWORK. UNDERSTAND IT WELL FOR YOUR NEXT EXAM.

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<th>Problem</th>
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(1) (a) Suppose $X_1, X_2, \ldots, X_n$ are independent Poisson random variables each having mean $\theta$. Determine the maximum likelihood estimator of $\theta$.

(b) The number of traffic accidents in Berkeley, California, in randomly chosen non-rainy days in 2010 is as follows:

3, 1, 5, 2, 4, 1, 2, 2

Use these data to estimate the proportion of non-rainy days that had not more than 2 accidents that year.
(2) Assume that the yield per acre for a particular variety of soybeans is $N(\mu, \sigma^2)$. For a random sample of $n = 5$ plots, the yields in bushels per acre were 37.4, 48.8, 46.9, 55.0 and 44.0.

(a) Give a point estimate for $\mu$.

(b) Find a 90% confidence interval for $\mu$. 
(3) The length of life of brand X light bulbs is assumed to be $N(\mu_X, 784)$. The length of life of brand Y light bulbs is assumed to be $N(\mu_Y, 627)$ and independent of X. If a random sample of $n = 56$ brand X light bulbs yielded a mean of $\bar{x} = 937.4$ hours and a random sample of size $m = 57$ brand Y light bulbs yielded a mean of $\bar{y} = 988.9$ hours, find a 90% confidence interval for $\mu_X - \mu_Y$. 
A civil engineer wishes to measure the compressive strength of two different types of concrete. A random sample of 10 specimens of the first type yielded the following data (in psi)

\[
Type 1: \quad 3,250, \ 3,268, \ 4,302, \ 3,184, \ 3,266, \\
\quad 3,297, \ 3,332, \ 3,502, \ 3,064, \ 3,116
\]

whereas a sample of 10 specimens of the second yielded the data

\[
Type 2: \quad 3,094, \ 3,106, \ 3,004, \ 3,066, \ 2,984, \\
\quad 3,124, \ 3,316, \ 3,212, \ 3,380, \ 3,018
\]

If we assume that the samples are normal with a common variance, determine

(a) a 95 percent two-sided confidence interval for \(\mu_x - \mu_y\), the difference in means;

(b) a 95 percent one-sided upper confidence interval for \(\mu_x - \mu_y\);

(c) a 95 percent one-sided lower confidence interval for \(\mu_x - \mu_y\).
(5) Let $Y$ be $b(100, p)$. To test $H_0 : p = 0.08$ against $H_1 : p < 0.08$, we reject $H_0$ and accept $H_1$ if and only if $Y \leq 6$.

(a) Determine the significance level $\alpha$ of the test.

(b) Find the probability of the Type 2 error if in fact $p = 0.04$. 
(6) Let $Y$ be $b(192, p)$. We reject $H_0 : p = 0.75$ and accept $H_1 : p > 0.75$ if and only if $Y \geq 152$. Use the normal approximation to determine

(a) $\alpha = P(Y \geq 152; p = 0.75)$.

(b) $\beta = P(Y < 152)$ when $p = 0.80$. 