Remember that this course is as much about explaining your answers clearly as it is about finding the correct answer. Unless otherwise indicated, at least 50 per cent of the credit for each question will be for clear explanations; you will not score very highly if you just write down the answers (especially if they are incorrect). Make sure you know and understand the main definitions, properties and methods of proof. For example, you should know the difference between propositions and propositional functions, how to prove statements of the form $P \Rightarrow Q$ and $P \iff Q$, the difference between elements and subsets of a set, how to prove $A \subseteq B$ and $A = B$, the definitions of special sets such as $\emptyset, \mathbb{Z}$ and $\mathbb{R}$, the definition of function, how to prove that a function is injective or surjective, the definitions of domain, codomain, range, image, cartesian product and graph, and the special functions like the floor and ceiling functions...among other things!! For extra practice, remember to try some of the odd (and even) number questions from the text.

(1) Find an examples of functions $f : \mathbb{Z} \to \mathbb{Z}$ and sketch their graphs, such that

(a) $f$ is neither surjective nor injective.
(b) $f$ is surjective but not injective.
(c) $f$ is injective but not surjective.
(d) $f$ is bijective.

(2) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be **strictly increasing** if

$$\forall x \forall y (x < y \to f(x) < f(y)).$$

(a) Which of the following are logically equivalent to the definition of a strictly increasing function. (You do not need to explain.)

(i) $\forall x (f'(x) > 0)$
(ii) $\forall x \forall y (x > y \to f(x) > f(y))$
(iii) $\forall x \forall y (x \geq y \to f(x) \geq f(y))$
(iv) $\forall x \forall y (f(x) \geq f(y) \to x \geq y)$
(v) $\forall x \exists y (x < y \land f(x) < f(y))$

(b) Prove that a strictly increasing function is one-to-one (injective). Use logical arguments, and NOT a result from calculus such as the horizontal line test.

(c) Find an example of a strictly increasing function which is not onto (not surjective). (Hint: It may be helpful to picture the graphs of some calculus functions, but you must explain your answer logically).

(3) Prove that $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$.

(4) Prove that $[\exists x P(x) \to \forall x Q(x)] \Rightarrow \forall x[P(x) \to Q(x)]$. Show that the converse statement is not true with an example.

(5) Write down the definition of surjective function $f : A \to B$ using quantifiers and logical symbols.