(1) **Abelian Groups revisited:** A binary operation on a set $A$ is a function $f : A \times A \rightarrow A$.

Rewrite the axioms for an abelian group, using the function $f$ instead of the symbol $+$ for the binary operation of addition. Make sure you understand the difference between $\forall x \exists y$ and $\exists y \forall x$.

(a) $\forall a \forall b (f(a, b) \in A)$
(b) $\forall a \forall b \forall c (f(f(a, b), c) = f(a, f(b, c)))$
(c) $\forall a \forall b (f(a, b) = f(b, a))$
(d) $\exists 0 \forall a (f(a, 0) = a)$
(e) $\forall a \exists b (f(a, b) = 0)$

(2) **Bijections between Intervals:**

(a) $f : (0, 1) \rightarrow (a, b), \text{ defined by } f(x) = (b - a)x + a$ and $f^{-1}(x) = \frac{x - a}{b - a}$.

(b) $f : (-1, 1) \rightarrow (-\infty, \infty), \text{ defined by } f(x) = \tan(\frac{\pi x}{2})$ and $f^{-1}(x) = \frac{2\tan^{-1}(x)}{\pi}$.

(c) $f : (a, b) \rightarrow (-\infty, \infty), \text{ defined by } f(x) = \tan(\pi \frac{2x - (a+b)}{2(b-a)})$

(d) $f : (0, 1) \rightarrow B = (a, \infty), \text{ defined by } f(x) = \tan(\frac{\pi x}{2}) + a$.

(3) If $f(x) = f(y)$ then $g(f(x)) = g(f(y))$ (evaluating the function $g$ at the same point). But then $x = y$ because $g \circ f$ is injective, and so $f$ is also injective, because $f(x) = f(y) \Rightarrow x = y$.

(4) $f : \mathbb{Z} \rightarrow \mathbb{R}$ defined by $f(a) = a$ and $g : \mathbb{R} \rightarrow \mathbb{Z}$ defined by $g(b) = \lfloor b \rfloor$. Then $g$ is surjective and $g(f(a)) = a$ so $g \circ f$ is surjective (even bijective, it is the identity function for $\mathbb{Z}$). But $f$ is not surjective, for example $f(a) = .5$ has no solution for $a$.

(5)

$$\lfloor -x \rfloor = n \iff n \leq -x < n + 1$$
$$\iff -(n + 1) < x \leq -n$$
$$\iff \lceil x \rceil = -n$$
$$\iff -\lceil x \rceil = n$$

so $\lfloor -x \rfloor = -\lceil x \rceil$. The other one is similar.

(6) Make sure your function only takes integer values. For $x < 1$ it looks like steps going down towards $-\infty$, for $x > 1$ the function values are all 0.

(7) Integer points on an upside down parabola.

(8) $f$ is not a function because $1/2 = 2/4$ and $2/3 = 6/9$ but $f(1/2, 2/3) = 3/5$ whereas $f(2/4, 6/9) = 8/13$. Functions are supposed to give exactly one output for each input. The problem is that rational numbers can be written in more than one way, so you have to be careful defining functions for $\mathbb{Q}$. 
