You must show all your work and explain your answers carefully. There is no credit for just writing down the answer, whether it is correct or not.

(1) **Tautologies:** Prove that the following compound propositions are tautologies (that is, they are always true no matter what the truth values of the propositions that occur in them)

- \([\neg p \land (p \lor q)] \rightarrow q\]
- \([(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)\]
- \([p \land (p \rightarrow q)] \rightarrow q\]

(a) Firstly, by using truth tables.

(b) Secondly, by showing they are logically equivalent to \(T\) using the properties in Tables 5 and 6 (p.17-18) (See example 6 on p.18)

(2) Use logical reasoning (and not truth tables) to explain why \((p \rightarrow q) \land (q \rightarrow r) \land (r \rightarrow p) \iff p \leftrightarrow q \leftrightarrow r\)

(3) Is “This statement is false” a proposition? Explain.

(4) **Knights and Knaves:** There are two kinds of inhabitant on an island: Knights, who always tell the truth, and Knaves, who always lie. There is no way to distinguish between knights and knaves visually. You are on the island and have the following encounters with the native inhabitants.

(a) You meet two people, A and B. A says “At least one of us is a knave” and B says nothing. Can you determine what A and B are, and if not, can you draw any conclusions?

(b) You then meet two more people, C and D. C says “We are both knights” and D says “C is a knave”. Can you determine what C and D are, and if not, can you draw any conclusions?

(c) You meet another two people who are guarding two doors, one is a knight and one is a knave (but you don’t know which is which). If you go through one door, you never have to take another math class again but if you go through the other door you are doomed to take math classes for another five years. You may ask ONLY one question and to ONLY one of the guards. What question should you ask to ensure that you choose the correct door, and never have to take another math class?

(d) You meet one person. You may ask ONE question to determine whether the person is a knight or knave, provided it is not a question you already know the answer to (for example, “Is my hair brown?” or “Is 1+1=2?”). What question would you ask?