You must show all your work and explain your answers carefully. There is no credit for just writing down the answer, whether it is correct or not.

(1) (a) Prove that a graph $G$ is bipartite if and only if every cycle of $G$ has even length.
(b) Prove that $Q_n$ is bipartite for every $n$.

(2) A graph is \textit{planar} if you can draw it in the plane (in 2 dimensions) so that no edges cross each other except at vertices.
(a) Show that $Q_3$ is planar.
(b) Show that $Q_4$ is planar.
(c) Given that $K_{3,3}$ is not planar, show that $Q_n$ is not planar for $n \geq 5$. (Hint: If a graph is planar, then so is any subgraph. Explain)

(3) Show that if $G$ is a bipartite simple graph with $v$ vertices and $e$ edges, then $e \leq v^2/4$. (Hint: calculus)

(4) How many paths are there from 101 to 010 in $Q_3$?

(5) Prove that a tree is a bipartite graph.