Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You will NOT receive full credit for a correct answer without explanation. No calculators.

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(1) (20 points) State whether the following are TRUE or FALSE. (No explanation required.)

(a) The cycle $C_3$ is a bipartite graph.

(b) $C_4$ is a bipartite graph.

(c) The complete graph $K_4$ is a bipartite graph.

(d) The 3-cube $Q_3$ is a bipartite graph.

(e) $K_3$ has an Euler circuit (a simple circuit which uses every edge.)

(f) $K_4$ has an Euler circuit.

(g) $Q_3$ has an Euler circuit.

(h) The complete bipartite graph $K_{3,3}$ is a planar graph.

(i) $Q_3$ is a planar graph.

(j) $K_{2,4}$ is a planar graph.
(2) (16 points) Let $G = (V, E)$ be a simple graph. $G$ is said to be self complementary if $G$ is isomorphic to its complementary graph $\overline{G}$. Recall that two vertices are adjacent in $G$ if and only if they are not adjacent in $\overline{G}$.

(a) Find a self complementary graph with 4 vertices. Explain. (Hint: Count edges)

(b) Prove that there is no self complementary graph $G$ with 6 vertices. (Hint: What is $G \cup \overline{G}$?)
(3) (16 points) The set $S$ is defined recursively by

1. $1 \in S$
2. $x, y \in S \Rightarrow x + 2y \in S$.

Prove that $S$ is the set of positive, odd integers.
(4) (16 points) Let $A = \{1, 2, 3, 4\}$.

(a) How many equivalence relations are there on $A$? Explain.

(b) How many equivalence relations $R$ are there on $A$ with the property that $(1, 2) \in R$. Explain.
(5) (16 points) If \( f : A \rightarrow A \) is a function, then \( G_f = \{(a, f(a)) | a \in A\} \) is a subset of \( A \times A \). Therefore \( G_f \) defines a relation on \( A \), and consequently an associated directed graph.

(a) Suppose that \( f \) is a \textbf{bijective} function. What property do the vertices and edges of the associated directed graph have? Explain.

(b) Suppose that \( G_f \) defines a reflexive relation on \( A \). What property must the function \( f \) have? Explain.
(6) (16 points) A deck of cards contains 52 cards, which are split into 4 suits (hearts, clubs, diamonds, spades). Within each suit there are 13 denominations 2, 3, … 10, J (= 11), Q (= 12), K (= 13), A (= 1 or 14).

A poker hand consists of 5 cards, and it does not matter which order you receive the cards, you may rearrange them in any way to make your best hand. A straight in poker consists of 5 cards which you can put in sequence where the cards may or may not be from different suits. For example 3, 4, 5, 6, 7, 8 and 8, 9, 10, J, Q and A, 2, 3, 4, 5 and 10, J, Q, K, A are straights but Q, K, A, 2, 3 is not a straight.

(a) How many different poker hands are there?

(b) How many poker hands are full houses? (A full house is 3 cards of one denomination, and 2 cards of another denomination. For example 2, 2, 2, J, J.)

(c) How many poker hands are flushes? (A flush is 5 cards which are all from the same suit but which is not a straight. For example 2, 4, 7, 10, K all from the hearts suit.)