Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do not want to have Problem 6 graded, you MUST put an “X” in the Points section of Problem 6.

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(1) (Page 119, Problem 29) A 2000 Mercedes sedan costs $47,275 and the car depreciates (loses value) a total of 47% during its first 5 years.

(a) If the depreciation is **linear**, find a formula for the value, \( V(t) \), of the car \( t \) years since 2000.

(b) If the depreciation is **exponential**, find a formula for the value, \( V(t) \), of the car \( t \) years since 2000.
(2) (Worksheet II) Use properties of logarithms to write the expression below as a **single** logarithm.

\[ 3 \ln(y) - \ln(K) - \frac{1}{5} \ln(x + 4) + \frac{1}{2} \ln(2x + 1) \]
(3) (Worksheet II)
(a) Find the exact solution(s) of \(2^{5x+3} = 3^{2x+1}\).

(b) Find the exact solution(s) of
\[
\log (y + 7) + \log (y - 11) = \log (14y - 37).
\]
(4) (Page 138, Problems 5 through 8) Let \( y = \ln(x + 1) - 2t \).

(a) Graph \( y \) against \( t \) for a positive constant \( x \). Label all intercepts and asymptotes (if they exist). Your answer may contain \( x \).

(b) Graph \( y \) against \( x \) for a positive constant \( t \). Label all intercepts and asymptotes (if they exist). Your answer may contain \( t \).
(5) (Page 138, Problem 14)

(a) Find the domain of

\[ f(x) = \ln(\ln(x)). \]

(b) Let \( H(x) = 3 \cdot e^{-3x} \). What happens to \( H \) as \( x \to \infty \)?
(6) (Sample Exam) Solve for $T$. You must show work.

$$P = a + \log \left( \frac{b}{c + T} \right)$$
(7) (Worksheet III) Answer the following questions:

(a) Find the percent annual growth rate of a quantity that triples in size every 13 years.

(b) Find the percent annual growth rate of a quantity that decreases by 6% per month.

(c) Suppose the price of a certain item is increasing due to inflation. If \( P = b(t) \) gives the price (in dollars) of the item \( t \) years after 1985, then \( b(t) \) is well approximated by the formula

\[
b(t) = 138(3.9)^{t/9}.
\]

Describe the economic interpretation of \( a \) if \( b^{-1}(185) = a \).
(8) (Page 145, Problem 15) The quantity of a substance is diminishing exponentially so that $Q(t) = 9.4 \left( \frac{1}{3} \right)^{t/7}$, where $t$ is measured in years, and $Q(t)$ is measured in grams.

(a) What is the half-life of this substance?

(b) If $Q$ is re-written as $Q(t) = K b^t$, find $b$, and explain its physical significance.
(9) (Page 152, Problem 22) Oil leaks from a tank. At $t = 0$ there are 1250 gallons of oil in the tank. Each hour after that 7% of the oil leaks out.

(a) What percentage of the original oil has leaked out after 12 hours?

(b) At what \textbf{continuous} percentage growth rate is the oil leaking out?
(10) (Page 156, Problem 6) Sam deposits $6600 into an account that earns 5% annual interest compounded monthly. Jerry deposits $5900 into an account that earns 6% annual interest, compounded continuously. When, if ever, will their balances be equal?