Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do not want to have Problem 6 graded, you MUST put an “X” in the Points section of Problem 6.

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(1) (Page 298, Problem 22) Two planes leave Logan Airport in Boston at 11 am. The air traffic controller reports that they are traveling away from each other at an angle of 114°. The DC-10 travels 534 mph and the L-1011 travels 537 mph. How far are they apart at 11:50 am? You must show all of your work.
(2) (Page 333, Example 4) Let $f$ and $g$ be two functions whose graphs are drawn below.

(a) Solve for $t$.

$$f(g(t)) \cdot g(t) = 0$$

(b) For what values of $t$ is $f(g(t)) < 0$?
(3) (Sample exam) The American Condor has been declared to be an endangered species and a group of the birds have been transferred to a protected reserve. The population of the birds $t$ years after being moved is given by

$$P(t) = \frac{50(2 + 0.3t)}{(4 + 0.01t)}.$$ 

Determine whether each of the statements below is TRUE or FALSE. You need not show any work.

(a) Initially 50 condors were moved into the protected area.

(b) The largest possible population that the protected area can sustain is 30 times larger than the initial population.

(c) The largest possible population that the protected area can sustain is 1500 condors.

(d) $P^{-1}(a) = b$ means that $a$ years after being moved there are $b$ condors in the reserve.

(e) $P^{-1}(25) = 0$. 
(4) (Page 390, Problem 13) Give one possible formula for the polynomial graphed below.
(5) (Page 407, Problem 18) Which of the following are true statements about the function \( k \), shown below?

\[
k(x) = \frac{4}{(x-2)^2} + \frac{3}{(2-x)(x-2)} - 4
\]

Circle all correct answers. You need not show any work.

(a) The graph of \( k \) can be obtained by shifting the graph of \( y = \frac{1}{x^2} \) two units to the left and 4 units downwards.

(b) The graph of \( k \) can be obtained by shifting the graph of \( y = \frac{1}{x^2} \) two units to the right and 4 units downwards.

(c) The graph of \( k \) can be obtained by reflecting the graph of \( y = \frac{1}{x^2} \) across the \( x \)-axis and shifting it two units to the left and 4 units downwards.

(d) \( k \) has a vertical asymptote \( y = 2 \) and a horizontal asymptote \( x = -4 \).

(e) \( k \) has a vertical asymptote \( x = 2 \) and a horizontal asymptote \( y = -4 \).

(f) \( k \) has an \( x \)-intercept of \( -15/4 \).

(g) \( k \) has a \( y \)-intercept of \( -15/4 \).

(h) \( k \) has two \( x \)-intercepts.
(6) (Page 407, Problem 28) Find one possible formula for a rational function that has its only zero at $x = 3$. It has a vertical asymptote at $x = -2$ and $x = 5$. It has a horizontal asymptote at $y = 0$. Its y-intercept is $3/5$. 
The cruising speed $V$ of birds at sea-level (in meters/sec) is determined by the mass $M$ of the bird (in grams), and the surface $S$ of the wings exposed to air (in square meters). It is given by

$$V = 0.169 \sqrt{\frac{M}{S}}.$$ 

(a) Is $V$ a power function of $M$? Explain.

(b) Is $V$ a power function of $S$? Explain.

(c) The wing surface area of a Canadian goose is typically 12 times that of an American robin, whereas the mass of the goose is 70 times that of the robin. How does the cruising speed of the goose compare to that of the robin?

(d) Using the information given in part(c) and the fact that the mass and the cruising speed of the American robin are typically 85 grams and 9.7 meters/sec respectively, find the wing surface area of the Canadian goose.
Given a right triangle with a hypotenuse of length 2, let one of the angles of the triangle be $\theta$, where $0 < \theta < \pi/4$. Let $x$ be the length of the side that is adjacent to $\theta$ and is not the hypotenuse. Express the following quantities in terms of $x$. Your answer must not include and trigonometric functions.

(a) $\sin(2\theta)$

(b) $(\tan \theta)^2$

c $\sin(\arccos \frac{x}{2})$

(d) Express $\arccos(-x/2)$ in terms of $\theta$. 
(9) (Page 347, Problem 33 & Worksheet) Find a formula for $p^{-1}(w)$ given

$$p(x) = \left(\frac{5e^{3x} + 2}{2e^{3x} + 1}\right)^{1/2}.$$
(10) (Worksheet III)

(a) Solve for the exact value(s) of $x$ in the interval $[0, 2]$.

\[ \frac{5}{2} \sin(2\pi x) + \cos^2(2\pi x) = 2 \]

(b) Find the exact value of $\sin^{-1}(\sin(5))$.

Show your work.
Useful formulas
For a triangle with sides $a$, $b$, $c$ and angles $A$, $B$, $C$ opposite these sides, respectively.

- Law of Sines: 
  \[
  \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
  \]

- Law of Cosines: 
  \[
  c^2 = a^2 + b^2 - 2ab \cos C
  \]

- Double angle: 
  \[
  \sin(2t) = 2 \sin t \cos t \quad \cos(2t) = \cos^2 t - \sin^2 t
  \]

- Half angle: 
  \[
  \sin(u/2) = \pm \sqrt{\frac{1 - \cos(u)}{2}} \quad \cos(u/2) = \pm \sqrt{\frac{1 + \cos(u)}{2}}
  \]

- Sum/Difference: 
  \[
  \sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) \\
  \sin(\theta - \phi) = \sin(\theta) \cos(\phi) - \sin(\phi) \cos(\theta) \\
  \cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi) \\
  \cos(\theta - \phi) = \cos(\theta) \cos(\phi) + \sin(\theta) \sin(\phi)
  \]

- Vertex form of a quadratic function: 
  \[
  y = a(x - h)^2 + k
  \]